Name:______________________________

EXPONENTIAL EQUATIONS

SHOW ALL WORK AND JUSTIFY ALL ANSWERS.

Supplies needed: Pencil and highlighter/colored pencil/crayon

1. Solve $4^x = 7$.

   (a) Do we know of a power of 4 that equals 7? (or a number we can raise 4 to in order to get 7?)

      i. Try $x = 1$: $4^1 =$

      ii. Try $x = 2$: $4^2 =$

      iii. We know our power will be a number between 1 and 2.

   (b) How else can we solve for x?

   (c) Notice that x is in the power. If only we knew of a way to bring a power down...
   Oh wait! We do know how to do that, if our function is a logarithm. How can we turn our function into a logarithm?
   There are two ways:

   (1) We can apply the log function to both sides of our equation:
   $\log(4^x) = \log 7$

      (a) Highlight the x in the power.

      (b) Now we can use a property of logarithms:

      Recall: $\log_b M^p = p \log_b M$

      (c) Rewrite as $x \log 4 = \log 7$. Highlight the x.
      It may be helpful to use parentheses: $(x)(\log 4) = \log 7$

      (d) Solve for x. Remember that $\log 4$ is just a number. We can rewrite our equation as $(\log 4)(x) = \log 7$ since multiplication over the real numbers is commutative. (i.e. 3×6 is the same as 6×3)

      NOTE: LHS means left-hand side. RHS means right-hand side.

Our problem:	A problem we know how to solve:
$(\log 4)(x) = \log 7$	$5x = 19$
Divide by the number on LHS that is multiplied by x: $\dfrac{(\log 4)(x)}{(\log 4)} = \dfrac{\log 7}{\log 4}$	Divide by the number on LHS that is multiplied by x: $\dfrac{5x}{5} = \dfrac{19}{5}$
$x = \dfrac{\log 7}{\log 4}$	$x = \dfrac{19}{5}$

      (e) SOLUTION: $x = \dfrac{\log 7}{\log 4}$.

      (f) While $x = \dfrac{\log 7}{\log 4}$ is a perfect, exact answer, a decimal approximation is helpful for us to see if our answer is in between 1 and 2, as we determined at the beginning of the problem.
      SOLUTION: $x = \dfrac{\log 7}{\log 4} \approx 1.4036775$

   (2) Another way to solve: Rewrite as a logarithm

   Recall: $\log_b n = m \Leftrightarrow b^m = n$

      (a) Rewrite $4^x = 7$ as $\log_4 7 = x$.

      (b) Can our calculators simplify $\log_4$? Most calculators can simplify $\log_{10}$ and $\log_e = \ln$

(c) How can we change this logarithm with base 4 into a logarithm we can type into our calculator?

What about the Change of Base formula?

Change of Base: $\log_b m = \dfrac{\log_a m}{\log_a b}$

$\log_4 7 = x \Leftrightarrow x = \dfrac{\log_{\text{any base}} 7}{\log_{\text{any base}} 4}$

(d) Try base 10: $x = \dfrac{\log 7}{\log 4} \approx$

(e) Try base e: $x = \dfrac{\ln\_}{\ln\_} \approx$

(f) Do you get the same answer?

2. Solve $3^{x-1} = 5^x$

(a) This problem is a little harder to estimate a value for x.

(b) Let's solve the same way as before, using the first method (NOTE: the second method is more difficult here). Notice that the variable x is in the power for terms on both sides of the equation. We will apply the log function to both sides of our equation.

(c) We can apply the log function to both sides of our equation:
$\log\left(3^{(x-1)}\right) = \log\left(5^x\right)$

(d) Highlight the **(x-1)** and **x** in the powers.

(e) Now we can use a property of logarithms:

Recall: $\log_b M^p = p \log_b M$

(f) Rewrite as $(x-1)\log 3 = x \log 5$. Highlight the $x-1$ and x parts that came from the powers. It is *essential* to use parentheses.

(g) Remember that $(x-1)\log 3$ is the same as $\log 3(x-1)$ and $x\log 5$ is the same as $(\log 5)x$. Using this commutative property, let's rewrite the equation:
$\log 3(x-1) = (\log 5)x$

(h) Now, let's compare this problem to a less complicated problem we already know how to solve.

Our problem:	A problem we know how to solve:
$\log 3(x-1) = (\log 5)x$	$5(x-1) = 19x$
Distribute on LHS: $(\log 3)x - \log 3 = (\log 5)x$	Distribute on LHS: $5x - 5 = 19x$
Move x terms to one side & the number to the other: $(\log 3)x - (\log 5)x = \log 3$	Move x terms to one side & the number to the other: $5x - 19x = 5$
Factor an x from the terms on RHS: $x(\log 3 - \log 5) = \log 3$	Factor an x from the terms on RHS: $x(5-19) = 5$
Divide by the number on LHS that is multiplied by x: $\dfrac{x(\log 3 - \log 5)}{(\log 3 - \log 5)} = \dfrac{\log 3}{(\log 3 - \log 5)}$	Divide by the number on LHS that is multiplied by x: $\dfrac{x(5-19)}{(5-19)} = \dfrac{5}{(5-19)}$
$x = \dfrac{\log 3}{(\log 3 - \log 5)}$ This is an exact solution. You're done!	$x = \dfrac{5}{(5-19)}$

3. Solve $2^x = 7^{x-3}$

   (a) We can apply the log function to both sides of our equation:

   (b) Now we can use a property of logarithms:

   Recall: $\log_b M^p = p\log_b M$

   (c) Distribute on RHS (because the power was a binomial):

   (d) Move x terms to LHS & the number to the RHS:

   (e) Factor an x from the terms on LHS:

   (f) Divide by the number on LHS that is multiplied by x:

   (g) SOLUTION: $x = \dfrac{-3\log 7}{\log 2 - \log 7}$

4. Solve $9^{x+2} = 5^x$

SOLUTION: $x = \dfrac{-2\log 9}{\log 9 - \log 5}$ or $x = \dfrac{2\log 9}{\log 5 - \log 9}$

5. Solve $4^{x-3} = 7^x$

SOLUTION: $x = \dfrac{3\log 4}{\log 4 - \log 7}$

6. Solve $2^x = 5^{x+4}$

SOLUTION: $x = \dfrac{4\log 5}{\log 2 - \log 5}$