Laws of Exponents: Let a and b be positive numbers and let x and y be real numbers. Then,

- $b^x b^y = b^{x+y}$
- $\frac{b^x}{b^y} = b^{x-y}$
- $(b^x)^y = b^{xy}$
- $(ab)^x = a^x b^x$
- $\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}$
- $a^{-x} = \frac{1}{a^x}$
- $\frac{1}{a^{-x}} = a^x$
- $\left(\frac{a}{b}\right)^{-x} = \left(\frac{b}{a}\right)^x$

How can we clean up something like: $\left(\frac{6x^3y^2x^{-8}}{15x^5y^4y^{-4}}\right)^{-3}$

SUMMARY:
While you can apply the exponent rules in any order, as long as you do them correctly, here is a suggested process:

1. If the fraction is raised to a negative power, rewrite the fraction by flipping the numerator and denominator. Rewrite the outer power, but now it is positive.
   RULE: $\left(\frac{a}{b}\right)^{-x} = \left(\frac{b}{a}\right)^x$
2. Reduce any numbers in the fraction if possible. It may be helpful to factor each integer down to its prime factorization so you can see what cancels.
   RULE: $\frac{a \cdot b \cdot c}{b \cdot d} = \frac{a \cdot \not{b} \cdot c}{\not{b} \cdot d} = \frac{a \cdot c}{d}$
3. Move any terms with negative exponents. If a term has a negative exponent in the numerator, move it to the denominator, making the exponent positive.
   RULE: $a^{-x} = \frac{1}{a^x}$ and $\frac{1}{a^{-x}} = a^x$
4. Combine variables in the numerator if the same variable appears multiple times.
   RULE: $b^x b^y = b^{x+y}$
5. Reduce any powers of variables that appear in both the numerator and denominator. One way to think of this is to subtract the powers and put that variable with the difference of powers where the biggest power was.
   RULE: $\frac{b^x}{b^y} = b^{x-y}$
6. Once the inside has been simplified completely, apply the exponent on the outside.
   RULE: $\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}$

Ex: Simplify the expression. Assume that the variable represents a positive number so that no absolute value symbols are needed. Write answers without using negative exponents.

$$\left(\frac{6x^3y^2x^{-8}}{15x^5y^4y^{-4}}\right)^{-3}$$

1. We have a negative power so rewrite the fraction by flipping the numerator and denominator. Rewrite the outer power, but now it is positive.

$$\left(\frac{6x^3y^2x^{-8}}{15x^5y^4y^{-4}}\right)^{\textcircled{-3}} = \left(\frac{15x^5y^4y^{-4}}{6x^3y^2x^{-8}}\right)^{\textcircled{3}}$$

2. Reduce any numbers in the fraction if possible. It may be helpful to factor each integer down to its prime factorization so you can see what cancels.

   So we have $\left(\frac{3\cdot 5\cdot x^5y^4y^{-4}}{2\cdot 3\cdot x^3y^2x^{-8}}\right)^3 = \left(\frac{\cancel{3}\cdot 5\cdot x^5y^4y^{-4}}{2\cdot \cancel{3}\cdot x^3y^2x^{-8}}\right)^3 = \left(\frac{5x^5y^4y^{-4}}{2x^3y^2x^{-8}}\right)^3$

3. Move any terms with negative exponents. If a term has a negative exponent in the numerator, move it to the denominator, making the exponent positive.

   $$\left(\frac{5x^5y^4\textcircled{y^{-4}}}{2x^3y^2\textcircled{x^{-8}}}\right)^3 = \left(\frac{5x^5y^4\textcircled{x^{+8}}}{2x^3y^2\textcircled{y^{+4}}}\right)^3$$

4. Combine variables in the numerator if the same variable appears multiple times.

   Let's start with combining the x terms:

   $$\left(\frac{5\textcircled{x^5}y^4\textcircled{x^8}}{2x^3y^2y^4}\right)^3 = \left(\frac{5\textcircled{x^{5+8}}y^4}{2x^3y^2y^4}\right)^3 = \left(\frac{5\textcircled{x^{13}}y^4}{2x^3y^2y^4}\right)^3$$

   Now combine the y terms:

   $$\left(\frac{5x^{13}y^4}{2x^3\textcircled{y^2}\textcircled{y^4}}\right)^3 = \left(\frac{5x^{13}y^4}{2x^3\textcircled{y^{2+4}}}\right)^3 = \left(\frac{5x^{13}y^4}{2x^3\textcircled{y^6}}\right)^3$$

5. Reduce any powers of variables that appear in both the numerator and denominator. One way to think of this is to subtract the powers and put that variable with the difference of powers where the biggest power was.

   Let's start with combining the x terms:

   $$\left(\frac{5\textcircled{x^{13}}y^4}{2\textcircled{x^3}y^6}\right)^3 = \left(\frac{5\textcircled{x^{13-3}}y^4}{2y^6}\right)^3 = \left(\frac{5\textcircled{x^{10}}y^4}{2y^6}\right)^3$$

   Now combine the y terms:

   $$\left(\frac{5x^{10}\textcircled{y^4}}{2\textcircled{y^6}}\right)^3 = \left(\frac{5x^{10}}{2\textcircled{y^{6-4}}}\right)^3 = \left(\frac{5x^{10}}{2\textcircled{y^2}}\right)^3$$

   NOTE: The power of y in the denominator was bigger than the power of y in the numerator, so we will only have y terms in the denominator. We can approach this by subtracting, but putting the result where we had the larger power of y.

6. Once the inside has been simplified completely, apply the exponent on the outside.

   $\left(\frac{5x^{10}}{2y^2}\right)^3 = \left(\frac{5^3(x^{10})^3}{2^3(y^2)^3}\right) = \frac{125x^{30}}{8y^6}$