Math 1314 – College Algebra
Section 2.1-2.2 The Rectangular Coordinate Systems and Graphs/Linear Equations in One Variable

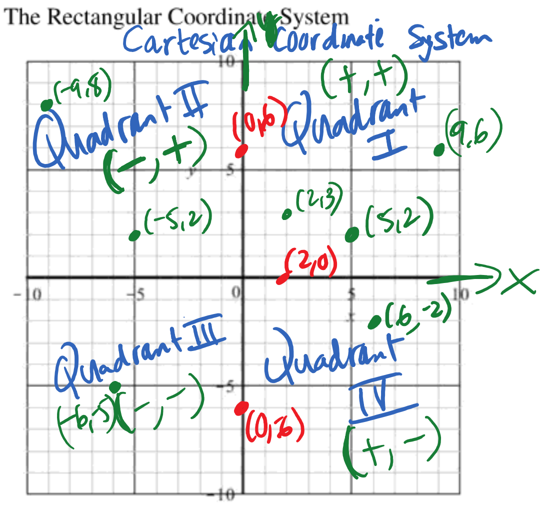
- The Rectangular Coordinate System

Cartesian Coordinate System

(x, y)

points on y-axis $(0, 6)$

points on x-axis $(a, 0)$



- Many important models are linear, which means the graph of the model is a straight line.

rate of change

- SLOPE of all non-vertical lines: $m = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$

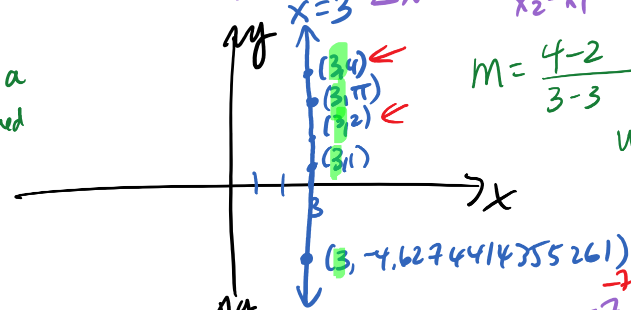
$\frac{N}{0}$ X

$\frac{0}{K} = 0$

$\frac{5}{0}$ $\frac{0}{5}$

- Vertical line:

have form $x = a$ and undefined slope.



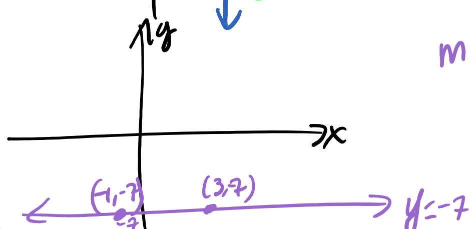
$m = \frac{4-2}{3-3} = \frac{2}{0}$ X

undefined

- Horizontal line:

$m = \frac{-7 - -7}{-1-3} = \frac{0}{-4} = 0$

Horiz lines have form $y = b$ and slope 0.



b is y-coordinate of y-int

NOTE: It does not matter which order you put the points in the formula, as long as you stay consistent.

- LINES: $y = mx + b$ slope-intercept form

$y - y_1 = m(x - x_1)$ point-slope form

$Ax + By = C$ / $Ax + By + D = 0$

Standard form / general form

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$m = \frac{(y - y_1)}{(x - x_1)}$

$m(x - x_1) = \frac{(y - y_1)}{\cancel{(x - x_1)}} \frac{\cancel{(x - x_1)}}{1}$

$m(x - x_1) = y - y_1$

$y - y_1 = m(x - x_1)$

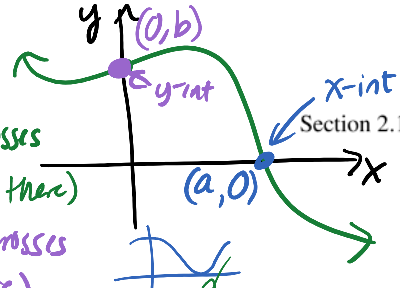
slope b/t $(1,2)$ & $(3,4)$

$m = \frac{4-2}{3-1} = \frac{2}{2} = 1$

or

$m = \frac{2-4}{1-3} = \frac{-2}{-2} = 1$

$a = b \Leftrightarrow b = a$



- x-intercept: point (a,0) that touches/crosses the x-axis. (y-coord is 0 there)

y-intercept: point (0,b) that touches/crosses the y-axis (x-coord is 0 there)

Ex: What are the x-intercept and y-intercept of the line $y = 3x - 5$? Sketch.

x-int: set $y=0$ $(5/3, 0)$

point

$0 = 3x - 5$

$+5 \quad +5$ Solve for x

$5 = 3x$

$\frac{5}{3} = \frac{3x}{3}$

$\frac{5}{3} = x$

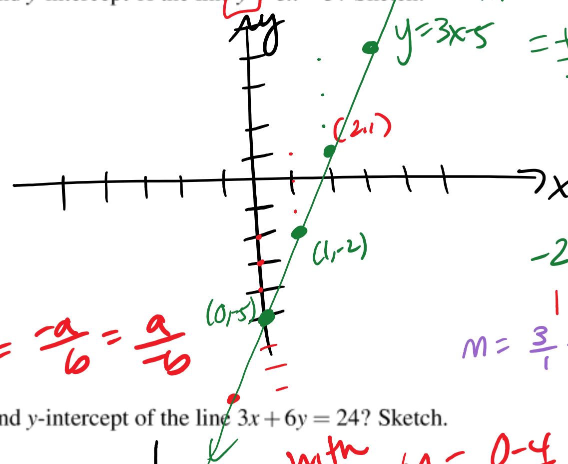
y-int: set $x=0$

$y = 3(0) - 5$

$y = -5$

point: $(0,-5)$

$-\frac{a}{6} = \frac{-a}{6} = \frac{a}{-6}$



$m = 3 = \frac{3}{1}$

$= \frac{+3}{+1}$ ← up 3 ← right 1

$-2 \overset{?}{=} 3(1) - 5$ ✓

$1 \overset{?}{=} 3(2) - 5$ ✓

$m = \frac{3}{1} = \frac{-3}{-1}$ ← down 3 ← left 1

Ex: What are the x-intercept and y-intercept of the line $3x + 6y = 24$? Sketch.

X-int: set $y=0$

$3x + 0 = 24$

$3x = 24$

$\frac{3x}{3} = \frac{24}{3}$

$x = 8$

$(8,0)$

y-int: set $x=0$

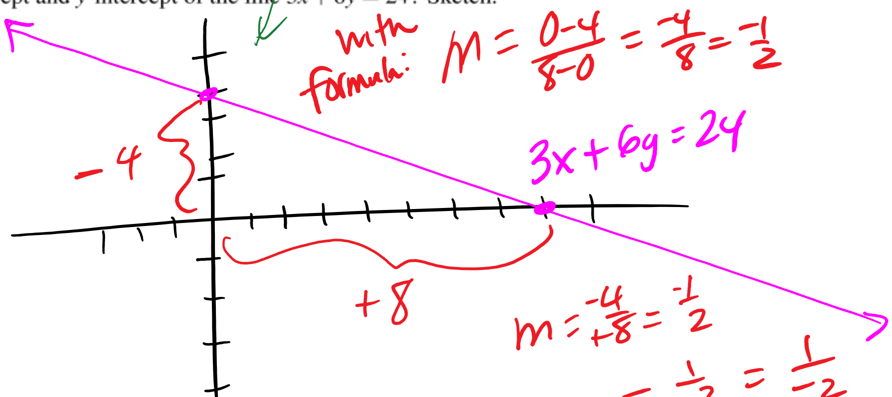
$0 + 6y = 24$

$6y = 24$

$y = 4$

$(0,4)$

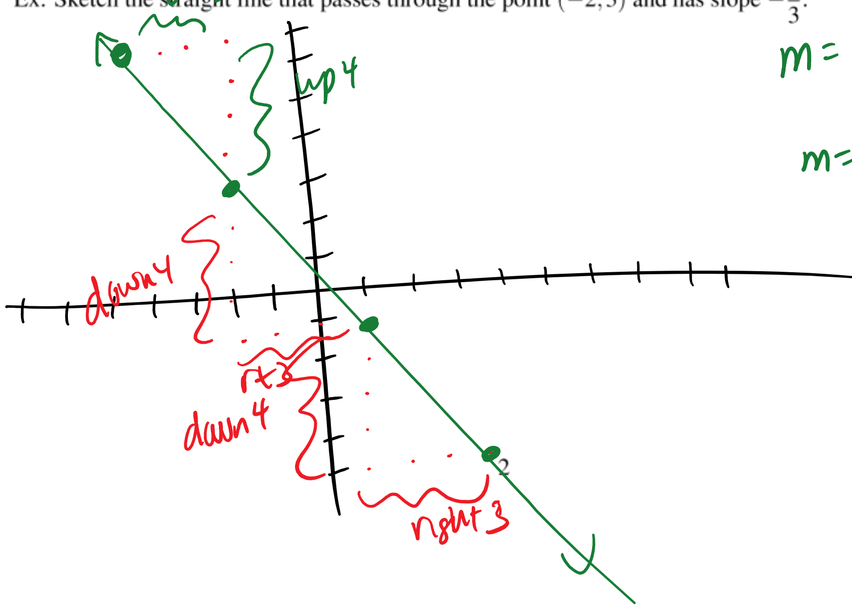
m with formula: $m = \frac{0-4}{8-0} = \frac{-4}{8} = \frac{-1}{2}$



$m = \frac{-4}{+8} = \frac{-1}{2}$

$-\frac{1}{2} = \frac{1}{-2}$

Ex: Sketch the straight line that passes through the point $(-2, 3)$ and has slope $-\frac{4}{3}$.



$m = -\frac{4}{3} = \frac{-4}{+3}$ ← down 4 ← right 3

$m = -\frac{4}{3} = \frac{+4}{-3}$ ← up 4 ← left 3 point-slope form: $y - y_1 = m(x - x_1)$

Ex: Find the equation of the line with slope $\frac{8}{3}$ that passes through the point (7,5).

$y - 5 = \frac{8}{3}(x - 7)$

$y - 5 = \frac{8}{3}x - \frac{8}{3}\left(\frac{7}{1}\right)$

$y - 5 = \frac{8}{3}x - \frac{56}{3}$
+5 +5

$y = \frac{8}{3}x - \frac{56}{3} + \frac{5}{1} \cdot \frac{3}{3}$

$y = \frac{8}{3}x - \frac{56}{3} + \frac{15}{3}$

$\boxed{y = \frac{8}{3}x - \frac{41}{3}}$

What is y-int? $(0, -\frac{41}{3})$

$m = \frac{9-2}{5-6}$

Ex: Find the equation of the line that passes through the points (5,9) and (6,2).

$m = \frac{2-9}{6-5} = \frac{-7}{1} = -7$

$y - y_1 = m(x - x_1)$

$y - 9 = -7(x - 5)$

$y - 9 = -7x + 35$
+9 +9

$\boxed{y = -7x + 44}$

What is y-int? (0,44)

Find eqn of line through (1,2) & (1,4)

$m = \frac{4-2}{1-1} = \frac{2}{0}$ X

Vertical line

$x = 1$

- Parallel lines: have the same slope & they never cross.

$L_1 \parallel L_2$ if $m_1 = m_2$

- Perpendicular lines: cross at a 90° angle & their slopes are negative reciprocals of each other.

$L_1 \perp L_2$ if $m_1 = -\frac{1}{m_2}$ (or if $m_1 m_2 = -1$)

Ex: Find the equation of the line that passes through the point (7,4) that is perpendicular to the line $10x + 11y = 5$.

Consider ⊥ line $10x + 11y = 5$.

Find slope. (Solve for y)

$11y = -10x + 5$

$y = \frac{-10}{11}x + \frac{5}{11}$

$m_\perp = \frac{-10}{11}$

Soooooooo... $m = \frac{11}{10}$

$y - y_1 = m(x - x_1)$

$y - 4 = \frac{11}{10}(x - 7)$

In standard form:

$10(y - 4) = \frac{11}{\cancel{10}}(x - 7)(\cancel{10})$

$10y - 40 = 11x - 77$
$-10y$ $-10y$

$-40 = 11x - 10y - 77$
+77 +77

$37 = 11x - 10y$

$\boxed{11x - 10y = 37}$ Std form

Ex: Find the equation of the line passing through (0,6) that is parallel to the line passing through (2,8) and (9,4).

∥ line: $m = \frac{8-4}{2-9} = \frac{4}{-7}$

$m_{\parallel} = -\frac{4}{7}$

$m = -\frac{4}{7}$

$y - 6 = -\frac{4}{7}(x - 0)$

$y - 6 = -\frac{4}{7}x$

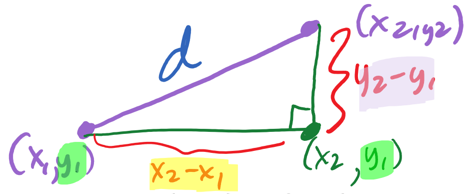
$\boxed{y = -\frac{4}{7}x + 6}$

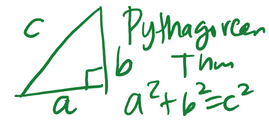
What is y-int? (0,6)

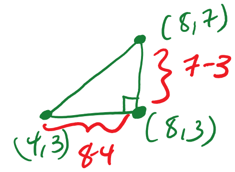
$y = mx + b$

$y = -\frac{4}{7}x + 6$

d (x_2, y_2) c Pythagorean







- Distance formula: The distance between points (x_1, y_1) and (x_2, y_2) is

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Ex: Find the distance between $(5,4)$ and $(9,18)$.

$$d = \sqrt{(9-5)^2 + (18-4)^2}$$
$$= \sqrt{4^2 + 14^2}$$
$$= \sqrt{16 + 196}$$
$$= \sqrt{212}$$
$$= \sqrt{2 \cdot 2 \cdot 53}$$
$$= 2\sqrt{53}$$

212
2 106
2 53

2, 3, 5, 7, 11, 13, 17, 19, 23,

- Midpoint formula: The midpoint of the line segment between points (x_1, y_1) and (x_2, y_2) is

$$M\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

Ex: If the midpoint of the line segment joining $P(-3,2)$ and $Q(x_2, y_2)$ is $M(1,4)$, find point Q. (NOTE: If points P and Q are on opposite sides of a circle passing through the center, then point M is the center of the circle.)

$$M(1,4) = \left(\frac{-3+x_2}{2}, \frac{2+y_2}{2}\right)$$

x: $(2)\,1 = \frac{-3+x_2}{\cancel{2}}\left(\frac{\cancel{2}}{1}\right)$

$2 = -3 + x_2$

$5 = x_2$

$\boxed{Q(5,6)}$

y: $(2)\,4 = \frac{2+y_2}{\cancel{2}}(\cancel{2})$

$8 = 2 + y_2$

$6 = y_2$