Math 1314 – College Algebra
Section 2.2 Linear Equations in One Variable – Part I

- Expression: any combination of operations, variables and numbers.
- Equation: A mathematical statement that sets two expressions equal to one another.
- Solution (aka: root): any number that satisfies an equation.
- Solution Set: the set of all solutions of an equation.
- Solve: The process used to find the solution set of an equation.

$\sqrt[\text{index}]{\text{radicand}}$

- Restrictions (i.e. values of the variable that are excluded from the set of possible solutions):
  - Cannot divide by 0.
  - Cannot have a negative under an even root
  - $\dfrac{\sim\sim}{\text{variable stuff}} \Rightarrow$ denominator $\neq 0$ $\quad \sqrt{7}$
  - $\sqrt[\text{even}]{\text{variable stuff}} \Rightarrow$ variable stuff ≥ 0 (radicand ≥ 0; $\geq$ non-negative)

Ex: Find the restrictions

(a) $\sqrt{x}+4=\dfrac{12}{y-17}$

LHS: radicand ≥ 0

$\boxed{x \geq 0}$

$\cancel{\sqrt{1}+4}$

RHS: frac w/ variable in denom

denom $\neq 0$

$y-17 \neq 0$

$\boxed{y \neq 17}$

(b) $3x-14=x+5$

no restrictions

(c) $\dfrac{8}{9}x+2=12$

No restrictions

We have fractions w/ variable in denom

(d) $\dfrac{3x}{x-1}-\dfrac{5}{x+3}=3$

denom $\neq$

$x-1 \neq 0$

$x \neq 1$

denom $\neq 0$

$x+3 \neq 0$

$x \neq -3$

$\cancel{x \neq 0}$

- Properties of Equality: If a,b,c are real numbers and $a=b$,

$a=b$	$a=b$	$a=b$	$a=b$
$a+c=b+c$	$a-c=b-c$	$\dfrac{a}{c}=\dfrac{b}{c}, c\neq 0$	$ac=bc$

- There are three types of equations:

Type of Equation	Definition	Example
Identity	True for all real values of the variable	$6x = 4x + 2x$ Soln: All real numbers
Conditional	True for only some values of the variable	$4x = 2x + 8$ Soln: $x = 4$
Contradiction or Inconsistent	No real number is a solution	$x - 2 = x$ Soln: No solution.

no restrictions

Ex: Solve $13x - 88 = 11 + 12$

$13x - 88 = 23$

$+88 \quad +88$

$\dfrac{13x}{13} = \dfrac{111}{13}$

$\boxed{x = \dfrac{111}{13}}$

Conditional equ

Ex: Solve no restrictions

(a) $16(1 + x) = 16(x + 2) - 11$

$16 + 16x = 16x + 32 - 11$

$16 + 16x = 16x + 21$

$-16 \quad -16$

$16x = 16x + 5$

$-16x \quad -16x$

$0 \neq 5$

NO SOLN

$16 + \cancel{16x} = \cancel{16x} + 21$

$-16x \quad -16x$

$16 \neq 21$

type of eqn: Contradiction

(b) $\dfrac{(x+1)}{(x-2)} = \dfrac{3}{(x-2)}$

Restrictions: denom $\neq 0$

$x - 2 \neq 0$

$x \neq 2$

LCM of denom: $(x-2)$

Multiply both sides by LCM of denom:

$\dfrac{(x+1)}{\cancel{(x-2)}} \cdot \dfrac{\cancel{(x-2)}}{1} = \dfrac{3}{\cancel{(x-2)}} \cdot \dfrac{\cancel{(x-2)}}{1}$

$x + 1 = 3$

extraneous soln $\rightarrow x = 2$

BUT WAIT!!

$x = 2$ was our restriction $x \neq 2$!!!

NO SOLN (b/c of restriction)

PEMDAS

(c) Solve for s: $P = L + \dfrac{si}{f}$

$-L \quad -L$

$f(P - L) = \dfrac{si}{\cancel{f}}(\cancel{f})$

$\dfrac{f(P-L)}{i} = \dfrac{s\cancel{i}}{\cancel{i}}$

$\boxed{\dfrac{f(P-L)}{i} = s}$

$7 = 3 + \dfrac{s(2)}{4}$

$-3 \quad -3$

$(4)\,4 = \dfrac{s(2)}{\cancel{4}}(\cancel{4})$

$16 = s(2)$

$\dfrac{16}{2} = \dfrac{2s}{2}$

$8 = s$

Restrictions: denom $\neq 0$, $x+3 \neq 0$, $x \neq -3$

denom $\neq 0$, $(x-1)(x+3) \neq 0$, $x-1 \neq 0$ AND $x-3 \neq 0$, $x \neq 1$, $x \neq -3$

Factor x^2+2x-3: $(x-1)(x+3)$

Signs in ( )( ) are different. Subtract to get $+2$. $+1 \; -3$, $-1 \; +3$

(d) $\dfrac{x+2}{x+3}+\dfrac{1}{x^2+2x-3}=1$

Restrictions: $x \neq 1$, $x \neq -3$

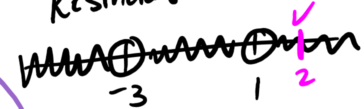
$$\frac{(x+2)}{(x+3)}+\frac{1}{(x-1)(x+3)}=1$$

- If $ab=0$, then $a=0$ or $b=0$
- If $ab \neq 0$, then $a \neq 0$ and $b \neq 0$

LCM of denominators: $(x+3)(x-1)$

$$\frac{(x+2)}{\cancel{(x+3)}}\cdot\frac{\cancel{(x+3)}(x-1)}{1}+\frac{1}{\cancel{(x-1)}\cancel{(x+3)}}\cdot\frac{\cancel{(x+3)}\cancel{(x-1)}}{1}=1(x+3)(x-1)$$

Restrictions:



$(x+2)(x-1)+1=(x+3)(x-1)$

$\cancel{x^2}-\cancel{x}+2x-2+1=\cancel{x^2}-\cancel{x}+3x-3$

$-x^2 \qquad -x^2$

$2x-1=3x-3$

$-2x \qquad -2x$

$-1=x-3$

$+3 \qquad +3$

$2=x$

Check this potential soln against our restrictions: $x \neq -3$, $x \neq 1$

We're good!

$x=2$ is soln

(e) $\dfrac{2x+3}{x^2+5x+6}+\dfrac{3x-2}{x^2+x-6}=\dfrac{5x-2}{x^2-4}$

Restrictions: denom $\neq 0$

$(x+2)(x+3) \neq 0$, $x \neq -2$ and $x \neq -3$

$(x-2)(x+3) \neq 0$, $x \neq 2$ and $x \neq -3$

$(x-2)(x+2) \neq 0$, $x \neq 2$ or $x \neq -2$

- x^2+5x+6, $(x+2)(x+3)$
  - signs in ( )( ) are same
  - use middle sign + add to get 5
  - $+1 \; +6$, $+2 \; +3$
- x^2+x-6, $(x-2)(x+3)$
  - Signs in ( )( ) are different
  - Subtract to get $+1$
  - $-1 \; +6$, $-2 \; +3$
- x^2-4, $(x+2)(x-2)$ Difference of 2 squares

$$\frac{(2x+3)}{(x+2)(x+3)}+\frac{(3x-2)}{(x-2)(x+3)}=\frac{(5x-2)}{(x-2)(x+2)}$$

LCM: $(x+2)(x+3)(x-2)$

$$\frac{(2x+3)}{\cancel{(x+2)}\cancel{(x+3)}}\cdot\frac{\cancel{(x+2)}\cancel{(x+3)}(x-2)}{1}+\frac{(3x-2)}{\cancel{(x-2)}\cancel{(x+3)}}\cdot\frac{(x+2)\cancel{(x+3)}\cancel{(x-2)}}{1}=\frac{(5x-2)}{\cancel{(x-2)}\cancel{(x+2)}}\cdot\frac{\cancel{(x+2)}(x+3)\cancel{(x-2)}}{1}$$

$(2x+3)(x-2)+(3x-2)(x+2)=(5x-2)(x+3)$

$2x^2-4x+3x-\cancel{6}+3x^2+6x-\cancel{2x}-4=5x^2+15x-\cancel{2x}-\cancel{6}$

$\cancel{5x^2}+5x-4=\cancel{5x^2}+15x$

$5x-4=15x$

$-5x \qquad -5x$

$-4=10x$

$\overline{10} \qquad \overline{10}$

$\frac{-4}{10}=x$

$\boxed{x=-\frac{2}{5}}$ Compare to restrictions: $x \neq -3$, $x \neq -2$, $x \neq 2$

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$$\frac{(2x+3)}{(x+2)(x+3)} + \frac{(3x-2)}{(x-2)(x+3)} = \frac{(5x-2)}{(x-2)(x+2)}$$

$$\frac{(2x+3)}{(\cancel{x+2})(\cancel{x+3})} \cdot \frac{(\cancel{x+2})(\cancel{x+3})(x-2)(x+3)(x-2)(x+2)}{1} + \frac{(3x-2)}{(x-2)(x+3)}$$

$$(2x+3)(x-2)(x+3)(x-2)(x+2) + (\quad)(\quad)(\quad)(\quad)$$