Math 1314 OS – College Algebra
Section 2.3 Models and Applications

- Strategy for Solving Word problems

1. Read the problem. Identify what information is given and what you are looking for. Draw a picture if possible.
2. Choose a variable to represent the quantity you are looking for. Express all other quantities in terms of this variable.
3. Express the variable in 2 different ways. May use a formula from geometry or physics or finance.
4. Set-up an equation relating the expressions in number 3.
5. Solve the equation.
6. Answer the questions in the problem.

Ex: In Jason's College Algebra class, homework counts for 5% of the final grade; quizzes for 10%; 3 exams each for 20%; and the final exam for 25%. If Jason has an 85 homework average, 62 quiz average, and scores of 75, 63 and 82 on his three exams, what does he need to make on the final exam to earn a 70 or higher in the course?

$.05HW + .1Q + .2E1 + .2E2 + .2E3 + .25F =$ Numerical average for course grade

$.05(85) + .1(62) + .2(75) + .2(63) + .2(82) + .25F = 70$

$4.25 \quad 6.2 \quad 15 \quad 12.6 \quad 16.4$

$54.45 + .25F = 70$

$-54.45 \quad -54.45$

$\frac{.25F}{.25} = \frac{15.55}{.25}$

$F = 62.2$

Jason needs at least a score of 63 on his final exam.

$ invest $10,000 in A ⇒ $15,000 in B
$20,000 in A ⇒ $5,000 in B
$1,000 in A ⇒ $24,000 in B
$a in A ⇒ $25,000 − a

$ invest $100 in A ⇒ earn $4 (.04)(100)
$200 in A ⇒ earn $8 (.04)(200)
$300 in A ⇒ earn $12 (.04)(300)
$1000 in A ⇒ $40 (.04)(1000)
$a in A ⇒ earn (.04)a

Ex: A retired couple invests $25,000, some in Bond A, which earns 4% and some in Bond B, which earns 6% annual interest. How much is invested at each rate if they earn $1300 each year?

a = amt invested in Bond A
25,000 − a = b = amt invested in Bond B

Earnings: (%)(amt invested) + (%)(amt invested) = total earned

.04a + .06(25,000 − a) = 1300
.04a + 1500 − .06a = 1300
−.02a + 1500 = 1300
−1500 −1500
−.02a = −200
÷ −.02 ÷ −.02
a = 10,000

frac: ① = ② reciprocal frac in numerator ③ multiply by reciprocal of frac in denom

$10,000 is invested in Bond A
$15,000 in Bond B.

Ex: A small mower can mow a pasture in 3 hours, and a larger mower can mow the pasture in 2 hours. How long will it take to mow the pasture if both mowers are used?

let x = # of hours to mow pasture using both

Small: can do 100% of job in 3hrs. ⇒ can do $\frac{1}{3}$ of job in 1 hr.
Larger: can do 100% of job in 2hrs ⇒ can do $\frac{1}{2}$ of job in 1 hr.
add these to get % of job w/ both
Both: can do 100% of job in x hrs ⇒ can do $\frac{1}{x}$ of job in 1hr.

$\frac{1}{3} + \frac{1}{2} = \frac{1}{x}$

LCM: 3(2)(x)

$\frac{1}{3}\frac{(3)(2)(x)}{1} + \frac{1}{2}\frac{(3)(2)(x)}{1} = \frac{1}{x}\frac{(3)(2)(x)}{1}$

2x + 3x = 6
5x = 6
$x = \frac{6}{5}$

$\frac{6}{5} = \frac{5}{5} + \frac{1}{5}$
1hr $\frac{1}{5}$ of hr $\frac{1}{5}$(60min)

It will take $\frac{6}{5}$ hrs. = 1hr 12min to mow pasture w/ both mowers.

NOTE: If there is a situation where one person or tool is working against the other, we subtract.

Ex: An empty pool can be filled in 10 hours. When full, the pool can be drained in 19 hours. How long will it take to fill the empty pool if the drain is left open?

x = # of hours together to "fill" the pool

Fill: Can do $\frac{1}{10}$ of job in 1hr
Drain: Can do $\frac{1}{19}$ of job in 1hr
Both: Can do $\frac{1}{x}$ of job in 1hr

$\frac{1}{10} - \frac{1}{19} = \frac{1}{x}$

LCM: (10)(19)(x)

$\frac{1}{10}\frac{(10)(19)(x)}{1} - \frac{1}{19}\frac{(10)(19)(x)}{1} = \frac{1}{x}\frac{(10)(19)(x)}{1}$

9x = 190
$x = \frac{190}{9} \approx 21.1$ hrs.

to fill pool w/ drain open.

LCM: $(10)(19)(x)$

$\frac{1}{10}\cdot\frac{(10)(19)(x)}{1} - \frac{1}{19}\cdot\frac{(10)(19)(x)}{1} = \frac{1}{x}\cdot\frac{(10)(19)(x)}{1}$

$19x - 10x = 190$

2

20% NaCl Soln

100 L of NaCl soln
of that 100L, 20L is NaCl.
How much is H_2O? 80L

Solute: NaCl
Solvent: H_2O

- A solution is made up of a solute dissolved in a solvent.
  Volume of solution X Concentration of Solute = Volume of Solute

(100 L)(.20) = 20

Ex: A nurse has 300 mL of a solution that is 30% alcohol. How much pure water must she add to dilute the solution down to 18% concentration?

x — no alcohol in Pure H_2O

.30

a(b+c) = abtac

(b+c)a

Solution 1 (Volume)(Conc of solute)	+	Solution 2 Pure H_2O (Volume)(Conc of solute)	=	New Solution (Volume)(Conc of solute)
(300 mL)(.30)	+	x (0)	=	(300+x)(.18)
90 mL	+	0 mL	=	(54+.18x) mL

$90 = 54 + .18x$

$-54 \quad -54$

$\frac{36}{.18} = \frac{.18x}{.18}$

$x = 200$ mL

$\frac{36}{\left(\frac{18}{100}\right)} = \frac{36}{1} \cdot \frac{100}{18} = 2 \cdot 100 = 200$

SOLUTION: 200 mL

Ex: A nurse has 300 mL of a solution that is 30% alcohol. How much pure alcohol must she add to increase the concentration of the solution to 38%?

100% → 1

Solution 1 (Volume)(Conc of solute)	+	Solution 2 (Volume)(Conc of solute)	=	New Solution (Volume)(Conc of solute)
(300mL)(.30)	+	(x)(1)	=	(300+x)(.38)
90	+	1x	=	114+.38x
−90		−.38x		−90 −.38x

$.62x = 24$

$\frac{.62x}{.62} = \frac{24}{.62}$

$x \approx 38.71$

SOLUTION: ≈ 38.71 mL

Ex: How many gallons of a 24% salt solution and 18% salt solution must be mixed to get 25 gallons of a salt solution with concentration 20%?

Solution 1 (Volume)(Conc of solute)	+	Solution 2 (Volume)(Conc of solute)	=	New Solution (Volume)(Conc of solute)
(x)(.24)	+	(25−x)(.18)	=	(25)(.20)

SOLUTION: ≈ 8.33 gallons

mixes Dist

rate = miles/hr = dist/time

$D = rt$ (miles/hr)(hr)

Ex: A man leaves home for a big work conference, driving at 70 mph. Ten minutes later, his wife realizes he left his briefcase, containing his cell phone and laptop, and follows him, traveling at 78 mph. How long will it take her to catch her husband?

	D	r	t
Man	d	70	$t + \frac{10}{60}$
wife	d	78	t

③ ② fill in constant col. ① start w/ variable

$d = d$

$d = r \cdot t$

man: $d = 70\left(t + \frac{1}{6}\right)$

wife: $d = 78t$

Solve

$70\left(t + \frac{1}{6}\right) = 78t$

$70t + \frac{70}{1}\left(\frac{1}{6}\right) = 78t$

$70t + \frac{70}{6} = 78t$

$\frac{1}{8} \cdot \frac{35}{3} = \frac{8t}{8}$

$1.46 \text{ hr} \approx t$

$\frac{6.5}{8.3} = \frac{8t}{8}$

For simplicity's sake, let's assume we purchase an item which is marked exactly 70% off, with an additional discount of exactly 30%.

Suppose we purchase an item that was originally \$100
Let's look at this in two steps:

$(1-.7)(100) = .3(100)$ $100\% - 70\% \Rightarrow$ Pay 30%

1. Take 70% off. How much do we pay? \$30
2. Now take 30% off of amount from Step 1. How much do we pay?

$(1-.3)(30)$
$(.7)(30) = \$21$

Let's try it again:
Suppose we purchase an item that was originally \$200

$(1-.7)(200) = .3(200)$

1. Take 70% off. How much do we pay? 60
2. Now take 30% off of amount from Step 1. How much do we pay?

$(1-.3)(60)$
$(.7)(60) = \$42$

SUMMARY: Suppose we purchase an item that was originally \$$x$

1. Take 70% off. How much do we pay? $(1-.7)x = .3x$
2. Now take 30% off of amount from Step 1. How much do we pay?

$(1-.3)(.3x)$

$(.7)(.3x) = .21x$