Math 1314 – College Algebra
Section 2.4 Complex Numbers

- Imaginary Numbers – based on the imaginary unit i, where $i = \sqrt{-1}$. Note: $i^2 = -1$

  $i^2 = (\sqrt{-1})^2 = -1$

- Complex numbers – numbers that can be written in the form $a + bi$, where a and b are real numbers and $i = \sqrt{-1}$. a is the real part, and b is the imaginary part.

  $(ab)^n = a^n b^n$

- Simplifying Imaginary numbers – follow the rules of exponents.

  Ex: Simplify (a) $(4i)^2$ $= 4^2 i^2$ $= 16(-1)$ $= -16$

  $i^2 = -1$

  $\sqrt{ab} = \sqrt{a}\sqrt{b}$

  (b) $\sqrt{-36}$ $= \sqrt{-1 \cdot 36}$ $= \sqrt{-1}\sqrt{36}$ $= i(6)$ $= 6i$

- NOTE: If a and b are both negative, then $\sqrt{ab} \neq \sqrt{a}\sqrt{b}$.

  Ex: Simplify $\sqrt{-25}\sqrt{-4}$

  $= 5i(2i)$ $= 10i^2$ $= -10$

  $i^2 = -1$

  $\cancel{\sqrt{(-25)(-4)} = \sqrt{+100} = 10}$

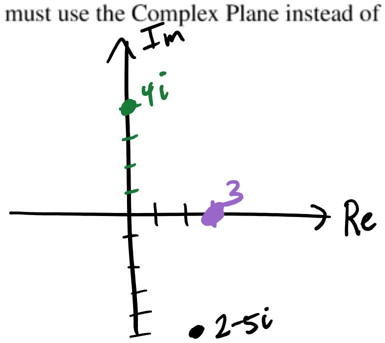
- Plotting a complex number: We must use the Complex Plane instead of the Cartesian Plane.

  Ex: Plot $2 - 5i$, 3, and $4i$.

  $2-5i$ Re: 2 Im: -5

  $4i = 0+4i$ Re: 0 Im: 4

  $3 = 3+0i$ Re: 3 Im: 0



- Two complex numbers are equal if their real parts are equal and their imaginary parts are equal.

  $a + bi = c + di$ if $a = c$ and $b = d$.

  real imaginary

  Ex: Find the values of x and y if $\underbrace{x + (x+y)i}_{\text{complex \#}} = \underbrace{3 + 8i}_{\text{complex \#}}$.

  real parts: $\boxed{x = 3}$

  Imaginary parts: $x + y = 8$, $3 + y = 8$, $\boxed{y = 5}$

Ex: Simplify $(5-2i)+(-3+9i)$.

$5-2i-3+9i$

$\boxed{2+7i}$

$(a+b)(a-b)=a^2-b^2$
Difference of 2 squares

Ex: Simplify (a) $(3+\sqrt{-36})(\cancel{7}-\sqrt{-16})$

$i^2=-1$

$(3+6i)(7-4i)$

FOIL:

$21-12i+42i-24i^2$

$-24(-1)$

$21+30i+24$

$45+30i$

real: 45 imag: 30

(b) ~~$(5+\sqrt{-16})(5-\sqrt{-16})$~~

$(5+\sqrt{-16})(5-\sqrt{-16})$

$i^2=-1$

$(5+4i)(5-4i)$

FOIL: $25-\cancel{20i}+\cancel{20i}-16i^2$

$25+16$

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Sq front $-$ Sq back

$25-16i^2$

$25+16$

- The complex numbers $a+bi$ and $a-bi$ are Complex conjugates of each other. In general, $(a+bi)(a-bi)=a^2-b^2i^2=a^2+b^2$

$i^2=-1$

Ex: Divide and write in $a+bi$ form: $\frac{3}{2-i}$

$i^2=-1$

$\frac{3}{(2-i)}\cdot\frac{(2+i)}{(2+i)}=\frac{6+3i}{4-i^2}=\frac{6+3i}{4+1}$

complex conjugates

Sq front $-$ Sq back

$a+bi$

real imag i

$=\frac{6+3i}{5}=\underbrace{\frac{6}{5}}_{\text{real}}+\underbrace{\frac{3}{5}}_{\text{imag}}i$

$\frac{a}{b}+\frac{c}{b}=\frac{a+c}{b}$

Ex: Simplify: $\frac{3+\sqrt{-25}}{2-\sqrt{-1}} = \frac{(3+5i)}{(2-i)}\frac{(2+i)}{(2+i)} = \frac{6+3i+10i+5i^2}{4-i^2}$

$i^2=-1$

$= \frac{6+13i-5}{4+1}$

$= \frac{1+13i}{5} = \boxed{\frac{1}{5}+\frac{13}{5}i}$

- Powers of i:

$i = \sqrt{-1} = i$
$i^2 = -1$
$i^3 = i^2 i = -1i = -i$
$i^4 = i^2 i^2 = (-1)(-1) = 1$ or $i^3 i = -i(i) = -i^2 = +1$
$i^5 = i^4 \cdot i = i$
$i^6 = i^4 \cdot i^2 = 1(i^2) = (1)(-1) = -1$
$i^7 = i^4 \cdot i^3 = 1(-i) = -i$
$i^8 = i^4 i^4 = 1(1) = 1$
$i^9 = i^8 i = 1i = i$
$i^{10} = i^9 i = i \cdot i = i^2 = -1$ or $i^8 i^2 = i^2 = -1$
$i^{11} = i^5 i^6 = i(-1) = -i$ or $i^8 i^3 = 1(-i) = -i$
$i^{12} = i^8 i^4 = 1$

$i^8 = (i^4)^2 = 1^2 = 1$
$i^{12} = (i^4)^3 = 1^3 = 1$

$i^{13} = i^{12} \cdot i = 1i = i$
$i^{14} = i^{12} \cdot i^2 = 1(-1) = -1$
$i^{15} =$

$i^{4n} = 1$

$i^{(\text{Multiple of } 4)} = 1$

$i^{400} = 1$

Ex: Simplify

(a) i^{43}

$= i^{40} i^3$
$= 1 \cdot i^3 = -i$

(b) $i^{365} = i^{364} i = i$

$= i^{360} i^5 = 1 \cdot i^4 \cdot i = i$

(c) $i^{440} = 1$

$i^2 = -1$

(d) i^{29}

$= i^{28} i = i$

(e) i^{18}

$= i^{16} i^2 = -1$

(f) $\frac{3}{i^5} = \frac{3}{i^4 \cdot i} = \frac{3}{i}\frac{(-i)}{(-i)}$

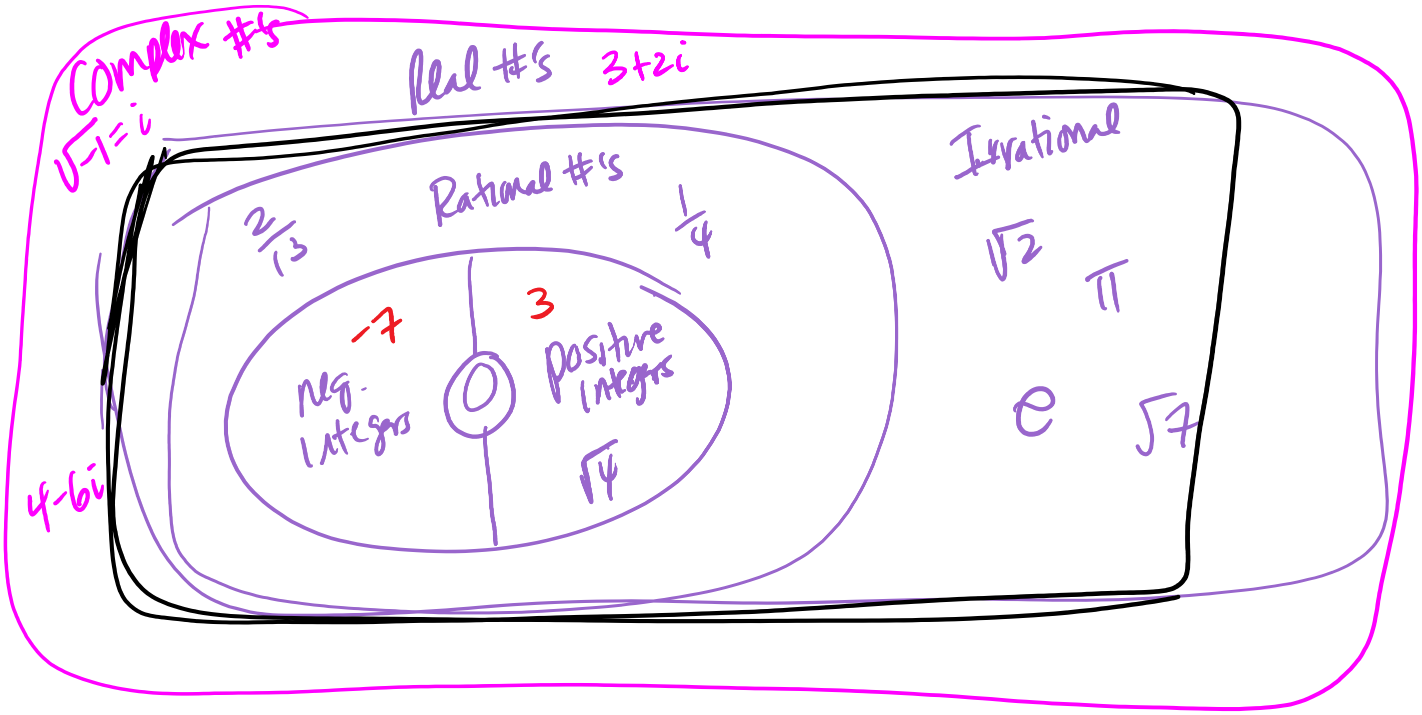
$= \frac{3}{0+i}\frac{(0-i)}{(0-i)}$

$= \frac{-3i}{-i^2}$

$= \frac{-3i}{-(-1)}$ +1

$= -3i$

real part: 0
imag part: -3


Complex #'s
$\sqrt{-1} = i$
Real #'s
$3+2i$
Rational #'s
$\frac{2}{13}$
$\frac{1}{4}$
Irrational
$\sqrt{2}$
π
-7
3
neg. Integers
0
positive Integers
$\sqrt{4}$
e
$\sqrt{7}$
$4-6i$