Math 1314 – College Algebra
Section 2.5 Quadratic Equations

- Quadratic Equation: Can be written in the form $ax^2 + bx + c = 0$ where a, b, c are real numbers and $a \neq 0$
- Zero Factor Theorem: If a and b are real numbers, and if $ab = 0$, then $a=0$ or $b=0$

$a=2$ $b=9$ $c=-35$ $ac=2(-35)=-70$

Ex: Solve (a) $2x^2 + 9x - 35 = 0$.

$-1\ \ +70$
$-2\ \ +35$
$-5\ \ +14$
$-7\ \ +10$

• Signs in (X) are different
• Subtract to get +9

$2x^2 - 5x + 14x - 35 = 0$

$x(2x-5) + 7(2x-5) = 0$

$(x+7)(2x-5) = 0$

$x+7=0$ or $2x-5=0$

$x=-7$ or $\frac{2x}{2} = \frac{5}{2}$

$x = \frac{5}{2}$

$\boxed{x=-7 \text{ or } x=\frac{5}{2}}$

(b) $6x^3 + 13x^2 - 8x = 0$

$a=6$ $b=13$ $c=-8$

$ac=-48$

$-1\ \ +48$
$-2\ \ +24$
$-3\ \ +16$
$-4\ \ +12$
$-6\ \ +8$

depressed eqn

$x(6x^2 + 13x - 8) = 0$

$x[6x^2 - 3x + 16x - 8] = 0$

$x[3x(2x-1) + 8(2x-1)] = 0$

$x(3x+8)(2x-1) = 0$

$\boxed{x=0}$ or $3x+8=0$ or $2x-1=0$

$3x=-8$ $\quad 2x=1$

$\boxed{x=-\frac{8}{3}}$ $\quad \boxed{x=\frac{1}{2}}$

- Square Root Property: If $c > 0$, the equation $x^2 = c$ has two real roots: $x = \pm\sqrt{c}$

$x=-\sqrt{c}$ or $x=+\sqrt{c}$

Ex: (a) Solve $x^2 - 4 = 0$.

$x^2 = 4$

$\sqrt{x^2} = \pm\sqrt{4}$

$x = \pm 2$

$(x+2)(x-2)=0$

$x=-2$ or $x=2$

(b) Solve $x^2 = 7$.

$x = \pm\sqrt{7}$

Ex: (a) Solve $x^2 - 27 = 0$.

$\sqrt{x^2} = \pm\sqrt{27}$

$x = \pm\sqrt{27}$

27
9 3
3 3

$\sqrt{3 \cdot 3 \cdot 3}$

$3\sqrt{3}$

$\boxed{x = \pm 3\sqrt{3}}$

(b) Solve $(x-11)^2 = 4$.

$\sqrt{(x-11)^2} = \pm\sqrt{4}$

$x - 11 = \pm 2$

$+11 \quad +11$

$x = 11 \pm 2$

$x = 11+2$ or $x = 11-2$

$x = 13$ $\quad x = 9$

$(a+b)^2 = a^2 + 2ab + b^2$

Sq front 2(front)(back) Sq back

$(x+b)^2 = |x^2 + 2bx + b^2$

- Completing the Square: $ax^2 + bx + c = 0$ (Remember: a, b, c may be positive, negative, or zero)
  1. If a, the coefficient of x^2, is not 1, change it to 1 by dividing both sides by a, the current coefficient of x^2.
  2. Move the constant to the right hand side.
  3. Complete the square on x:
     (a) Take one-half of the coefficient of x, and then square it. HINT: leave as $\left(\frac{b}{2a}\right)^2$
     (b) Add this number to both sides of the equation.
  4. Factor the perfect square trinomial. NOTE: It will always be $\left(x + \frac{b}{2a}\right)^2$.
     ***Keep the sign from 3(a) and combine like terms.
  5. Solve the resulting quadratic equation by square root property.

$(a+b)^2 = a^2 + 2ab + b^2$

$(a+(-b))^2 = (a-b)^2 = a^2 + 2(a)(-b) + (-b)^2$

$(x+b)^2 = x^2 + 2(bx) + b^2$

$a = 1$

Ex: Solve by completing the square: $x^2 - 2x - 9 = 0$.

$x^2 - 2x = 9$

$\frac{1}{2}(\text{coeff of } x) = \frac{1}{2}(-2) = -1$

$(x-1)^2 = (x-1)(x-1) = x^2 - x - x + 1 = x^2 - 2x + 1 = x^2 - 2x + (-1)^2$

$x^2 - 2x + (-1)^2 = 9 + (-1)^2$

$(x-1)^2 = 9 + 1$

$(x-1)^2 = 10$

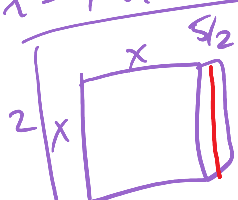
$\sqrt{(x-1)^2} = \pm\sqrt{10}$

$x - 1 = \pm\sqrt{10}$

$+1 \quad +1$

$\boxed{x = 1 \pm \sqrt{10}}$

$x = 1 + \sqrt{10} \qquad x = 1 - \sqrt{10}$



Ex: Solve by completing the square: $2x^2 + 5x = 12$.

$\frac{2x^2}{2} + \frac{5x}{2} = \frac{12}{2}$

$1x^2 + \frac{5}{2}x = 6$

$|x^2 + \boxed{2bx} + (b)^2$

$\frac{1}{2}\left(\frac{5}{2}\right) = \frac{5}{4}$

Sq it: $\left(\frac{5}{4}\right)^2$

$2bx = \frac{5}{2}x$

$\frac{2b}{2} = \frac{5}{2} \cdot \frac{1}{2}$

$x^2 + \frac{5}{2}x + \left(\frac{5}{4}\right)^2 = 6 + \left(\frac{5}{4}\right)^2$

$\left(x + \frac{5}{4}\right)^2 = \frac{6 \cdot 16}{1 \cdot 16} + \frac{25}{16}$

$\left(x + \frac{5}{4}\right)^2 = \frac{96}{16} + \frac{25}{16}$

$\left(x+\frac{5}{4}\right)\left(x+\frac{5}{4}\right) = x^2 + \frac{5}{4}x + \frac{5}{4}x + \left(\frac{5}{4}\right)^2$

$x^2 + 2\left(\frac{5}{4}\right)(x) + \left(\frac{5}{4}\right)^2$

$\sqrt{\left(x+\frac{5}{4}\right)^2} = \pm\sqrt{\frac{121}{16}}$

$x = -\frac{5}{4} \pm \frac{11}{4}$

$(x+\frac{5}{4})^2 = \frac{121}{16}$

$\sqrt{(x+\frac{5}{4})^2} = \pm\sqrt{\frac{121}{16}}$

$x + \frac{5}{4} = \pm\frac{11}{4}$

$x = -\frac{5}{4} \pm \frac{11}{4}$

$x = -\frac{5}{4} + \frac{11}{4}$ or $x = -\frac{5}{4} - \frac{11}{4}$

$x = \frac{6}{4} = \boxed{\frac{3}{2}}$ $\qquad$ $x = \frac{-16}{4} = \boxed{-4}$

$x^2 + 2(\frac{5}{4})(x) + (\frac{5}{4})^2$

$87 = 3 \cdot 29$

$\sqrt{-1} = i$

- The Quadratic Formula: The solutions of the general quadratic equation $ax^2+bx+c=0$ $(a \neq 0)$ are

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

Ex: Use the quadratic formula to solve $4x^2+13x+16=0$.

$a=4$, $b=13$, $c=16$

$$x = \frac{-13 \pm \sqrt{13^2-4(4)(16)}}{2(4)} = \frac{-13 \pm \sqrt{169-256}}{8} = \frac{-13 \pm \sqrt{-87}}{8} = \boxed{\frac{-13 \pm \sqrt{87}\,i}{8}} = \boxed{-\frac{13}{8} \pm \frac{\sqrt{87}}{8}i}$$

Complex Solns

- Discriminant: If a, b, and c are real numbers (and $a \neq 0$), then the discriminant is b^2-4ac. Used to determine the nature of the roots of an equation of the form $ax^2+bx+c=0$.

Discriminant	# and Type of Roots
0	One repeated rational root
Positive and a perfect square	Two different rational roots
Positive and not a perfect square	Two different irrational roots
Negative	No real number roots (Two complex roots)

$x = \frac{2 \pm \sqrt{0}}{4} = \frac{2+0}{4} = \frac{2-0}{4}$

$x = \frac{2 \pm \sqrt{16}}{4} = \frac{2 \pm 4}{4} = \frac{2+4}{4}$ or $\frac{2-4}{4}$

$x = \frac{4 \pm \sqrt{7}}{2}$

Ex: Determine the nature of the roots of $10x^2+x=21$. $\Rightarrow 10x^2+x-21=0$

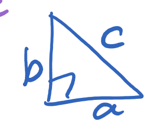
$a=10$, $b=1$, $c=-21$

Discriminant: b^2-4ac
$= 1-4(10)(-21)$
$= 1+840$
$= 841 = 29^2$ positive perfect square

$\boxed{\text{2 different rational roots}}$

- Pythagorean Theorem: For a right triangle with legs of length a and b and hypotenuse of length c:

$a^2+b^2=c^2$



Ex: A formula for the normal systolic blood pressure for a man age A, measured in mmHg, is given as $P = 0.006A^2 - 0.02A + 120$. Find the age to the nearest year of a man whose normal blood pressure measures 125 mmHg.

Find A when $P = 125$

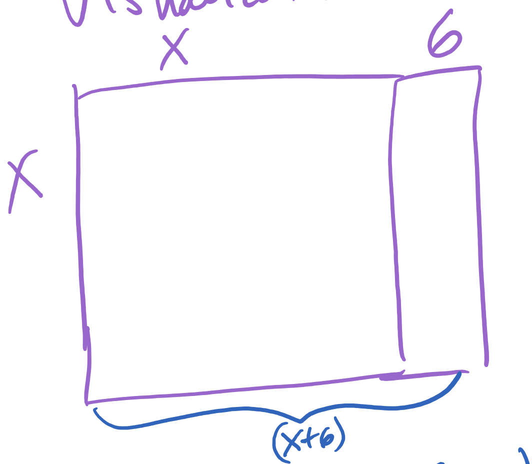
$a = .006$, $b = -.02$, $c = -5$

$125 = .006A^2 - .02A + 120$
$-125 \qquad\qquad -125$
$0 = .006A^2 - .02A - 5$

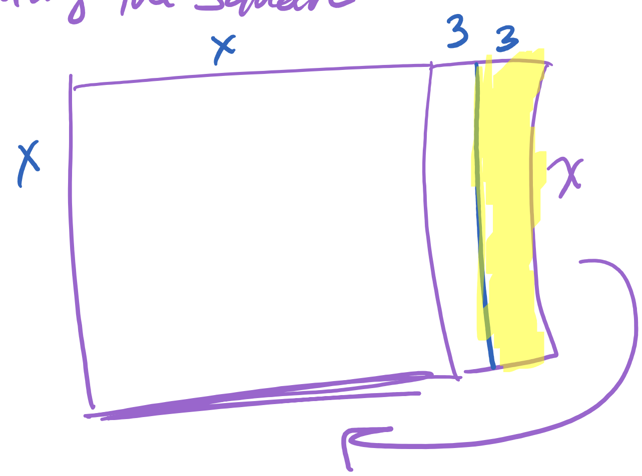
$$A = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{+.02 \pm \sqrt{(-.02)^2-4(.006)(-5)}}{2(.006)}$$

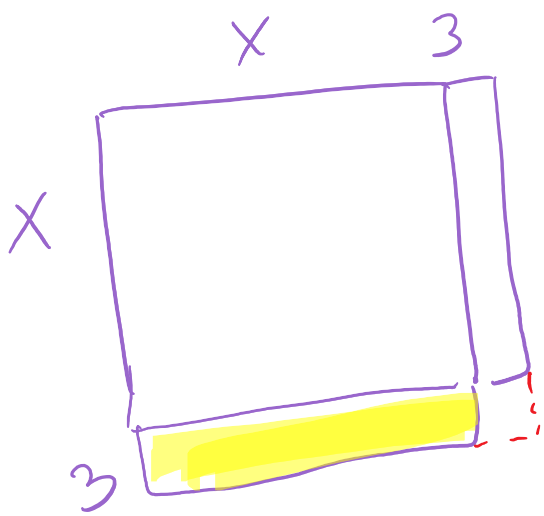
Man is approx 31 years old $A \approx 30.5823$ or ~~$A \approx -27$~~

Visualization of Completing the Square

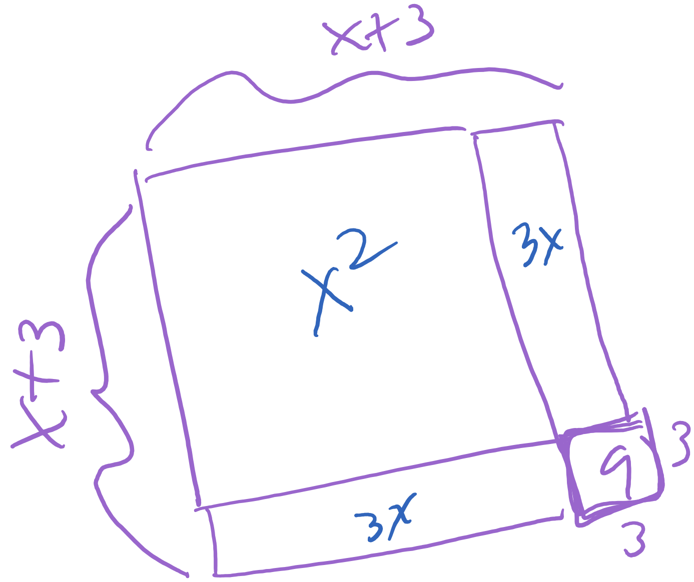


$A = x^2 + \underline{6}x = x(x+6)$





$A = x^2 + 6x$



$$x^2 + 6x + 9 = (x+3)^2$$