MATH 1314 – COLLEGE ALGEBRA
SECTION 2.6 OTHER TYPES OF EQUATIONS

- Recall: Rules of Exponents: If m, n are natural numbers:

  - $x^m x^n = x^{m+n}$
  - $(xy)^a = x^a y^a$
  - $(x^m)^n = x^{mn}$
  - $\left(\frac{x}{y}\right)^m = \frac{x^m}{y^m}$

- If $a \geq 0$ and n is a natural number, then $a^{1/n}$ (read as the nth root of a) is the non-negative real number such that $(a^{1/n})^n = a$.

- Rule for Rational Exponents: If m and n are positive integers, the fraction $\frac{m}{n}$ is in lowest terms, and $a^{1/n}$ is a real number, then $a^{m/n} = (a^{1/n})^m = (a^m)^{1/n}$

- If n is a natural number greater than 1 and if $a^{1/n}$ is a real number, then $\sqrt[n]{a} = a^{1/n}$

- In the radical expression $\sqrt[n]{a}$, the symbol $\sqrt{}$ is the radical sign, a is the radicand, and n is the index.

- Properties of Radicals
  Let x, y be real numbers and a, b, m, n be natural numbers. Let $\sqrt[n]{x}$, $\sqrt[n]{y}$ be real numbers.

  - Product Rule: $\sqrt[n]{xy} = \sqrt[n]{x}\sqrt[n]{y}$
  - Power Rule: $\sqrt[n]{x^m} = (\sqrt[n]{x})^m$
  - Quotient Rule: $\frac{\sqrt[n]{x}}{\sqrt[n]{y}} = \sqrt[n]{\frac{x}{y}}$
  - If n is odd, $\sqrt[n]{x^n} = x$. If n is even, $\sqrt[n]{x^n} = |x|$.

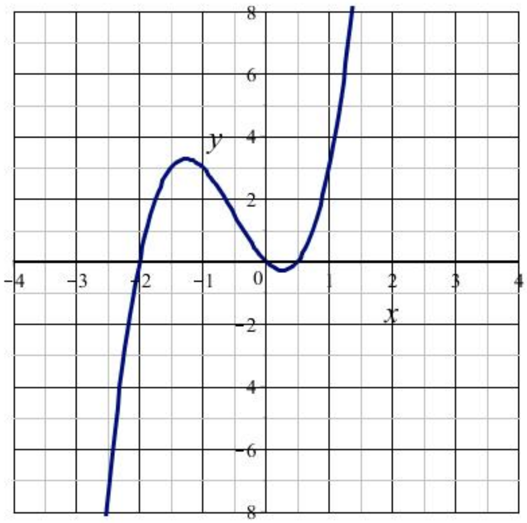
- Power Property of Real Numbers: If a and b are real numbers, n is an integer, and $a = b$, then $a^n = b^n$.
  Ex:

  NOTE: When we raise both sides of an equation to a power > 1,

  Ex: Solve $(x-3)^{3/2} = 64$

Ex: Solve $2x^3 + 3x^2 - 2x = 0$.

Ex: Solve $2m^{2/3} + 3m^{1/3} - 2 = 0$.



Ex: Solve $\sqrt{x-3} + 4 = 7$

Ex: Solve $\sqrt{2m+1} = -(1-m)$.

Ex: Solve $\sqrt{2x+1} = 6 - \sqrt{x+5}$.

Ex: Solve $p - p^{1/2} = 20$.

- The absolute value of a real number x, denoted $|x|$ is given by:

$$|x| = \begin{cases} -x & \text{if} \quad x < 0 \\ x & \text{if} \quad x \geq 0 \end{cases}$$

- The absolute value of a number represents

- Suppose $|x| = 5$. What values can x be?

- Suppose $|x| = 0$. What values can x be?

- Suppose $|x| = -7$. What values can x be?

- So the absolute value of any expression must return a ____________________ value.

- **SUMMARIZING:** Suppose $|x| = c$ where c is a real number then:

  - if $c > 0$ then there are two solutions:
  - if $c = 0$ then there is one solution:
  - if $c < 0$ then there are zero solutions.

Applying this concept we can solve similar equations.

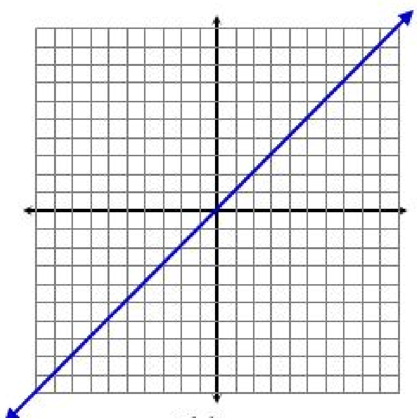
Ex: Solve.

(a) $|3x - 5| = 12$

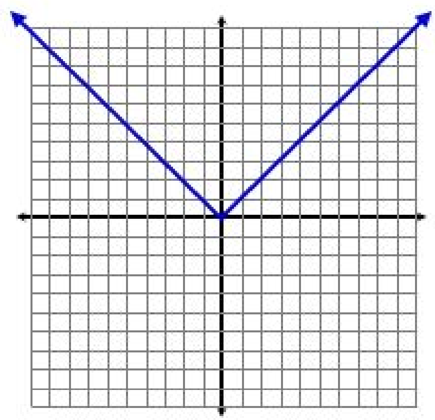
(b) $4|x + 1| - 3 = 18$

(c) $|2x+11| = -5$.

■ Consider the graphs of $f(x) = x$ and $g(x) = |x| = \begin{cases} -x & \text{if} \quad x < 0 \\ x & \text{if} \quad x \geq 0 \end{cases}$



$f(x) = x$



$g(x) = |x|$

■ How can the the graph of $f(x) = x$ be used to derive the graph of $g(x) = |x|$?