Math 1314 – College Algebra
Section 2.7 Linear Inequalities and Absolute Value Inequalities

- Properties of Inequalities:
  - Trichotomy Property: For all real numbers a and b, one of the following statements is true:
    $a<b$ or $b<a$ or $a=b$
  - Transitive Property: If a, b, and c are real numbers, then
    if $a<b$ and $b<c$, then $a<c$

    if $a>b$ and $b>c$, then $a>c$

  - Addition/Subtraction Properties: Let a, b, and c be real numbers. $a+c<b+c$
    - If $a<b$, then $a+c<b+c$
    - If $a<b$, then $a-c<b-c$
    - Similarly for $\leq, \geq, >$
  - Multiplication/Division Properties: Let a, b, and c be real numbers.
    1. If $a<b$ and $c>0$, then $ca<cb$
       If $a<b$ and $c>0$, then $\frac{a}{c}<\frac{b}{c}$
    2. If $a<b$ and $c<0$, then $ca>cb$
       If $a<b$ and $c<0$, then $\frac{a}{c}>\frac{b}{c}$

    c ↓ $ca<b$

    $2<4$

    $-2 > -4$

    NOTE: If we multiply or divide by a negative number, MUST flip the ineq. sign.
- We will solve quadratic and absolute value functions analytically and graphically.

  Ex: Let $f(x)=3x-1$ and $g(x)=2$.

  (a) Solve $f(x)=g(x)$.

  $3x-1=2$

  $3x=3$

  $x=1$

  (b) Solve $f(x)>g(x)$.

  $3x-1>2$

  $3x>3$

  $x>1$

  $(1,\infty)$

  (c) Solve $f(x)<g(x)$.

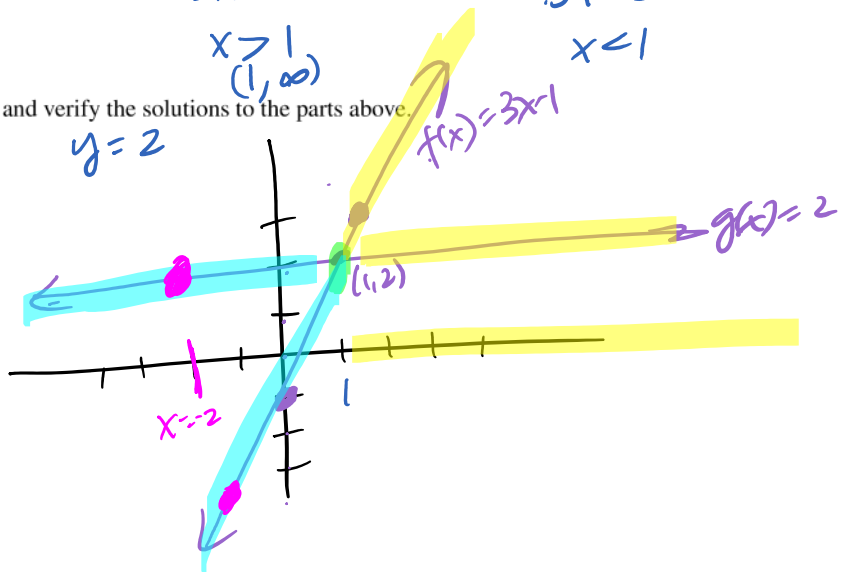
  $3x-1<2$

  $3x<3$

  $x<1$

  (d) Graph both functions and verify the solutions to the parts above.

  $y=3x-1$ $y=2$



- Solving Linear Inequalities:

Ex: Solve $5(x-4) > 25$.

$5x - 20 > 25$

$+20 \quad +20$

$5x > 45$

$\frac{5x}{5} > \frac{45}{5}$

$x > 9$

or

$\frac{5(x-4)}{5} > \frac{25}{5}$

$x - 4 > 5$

$+4 \quad +4$

$x > 9$

$(9, \infty)$

PEMDAS

Ex: Solve $-4(x+3) \geq 16$.

$-4x - 12 \geq 16$

$+12 \quad +12$

$-4x \geq 28$

$\frac{-4x}{-4} \leq \frac{28}{-4}$

$x \leq -7$

$(-\infty, -7]$

OR

$\frac{-4(x+3)}{-4} \leq \frac{16}{-4}$

$x + 3 \leq -4$

$x \leq -7$

- Compound Inequalities:

Ex: Solve $-5 \leq 2x + 1 < 9$

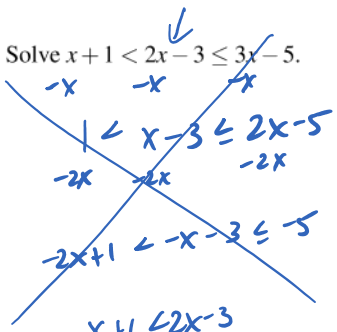
$-1 \quad -1 \quad -1$

$\frac{-6}{2} \leq \frac{2x}{2} < \frac{8}{2}$

$-3 \leq x < 4$

Interval notation: $[-3, 4)$

Ex: Solve $x + 1 < 2x - 3 \leq 3x - 5$.



$-x \quad -x \quad -x$

$1 < x - 3 \leq 2x - 5$

$-2x \quad -2x \quad -2x$

$-2x + 1 < -x - 3 \leq -5$

$x + 1 < 2x - 3$ AND $2x - 3 \leq 3x - 5$

$-x \quad -x \quad -2x \quad -2x$

$1 < x - 3 \quad -3 \leq x - 5$

$+3 \quad +3 \quad +5 \quad +5$

$4 < x \quad 2 \leq x$

$x > 4$ AND $x \geq 2$

$x + 1 < 2x - 3$

$-2x \quad -2x$

$-x + 1 < -3$

$-1 \quad -1$

$-x < -4$

$\frac{-x}{-1} > \frac{-4}{-1}$

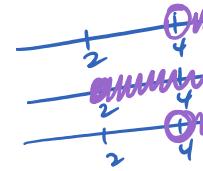
$x > 4$

$x > 4$:

AND

$x \geq 2$:

Soln:

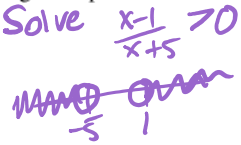


look for values shaded in both # lines

Interval notation: $(4, \infty)$

- Recap: For linear inequalities, we solve like an equation.
- For nonlinear inequalities:
  1. Arrange the inequality so that you are comparing an expression to zero.
  2. Replace the inequality sign with an = sign.
  3. Solve like an equation.
  4. Test intervals on a number line.

Solve $\frac{x-1}{x+5} > 0$



plug in $x=1$: $\frac{1-1}{1+5} \stackrel{?}{>} 0$ $\quad 0 \stackrel{?}{>} 0$ NO

$\frac{0}{6} > 0$

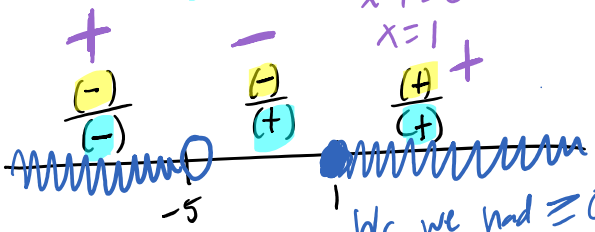
- Rational Inequalities: Remember to arrange the inequality so that you are comparing an expression to zero. Set numerator=0 and solve. Set denominator=0 and solve. Test each interval on a number line.

Ex: Solve $\frac{x-1}{x+5} \geq 0$

Numerator=0: $x-1=0$, $x=1$

Denom=0: $x+5=0$, $x=-5$, open circle

Never include a value from solving denom=0



b/c we had ≥ 0, include value that makes Num=0 (unless it also makes denom=0)

Soln: interval notation: $(-\infty, -5) \cup [1, \infty)$

- Recall: $|x|$ means dist b/t x & 0 on a # line

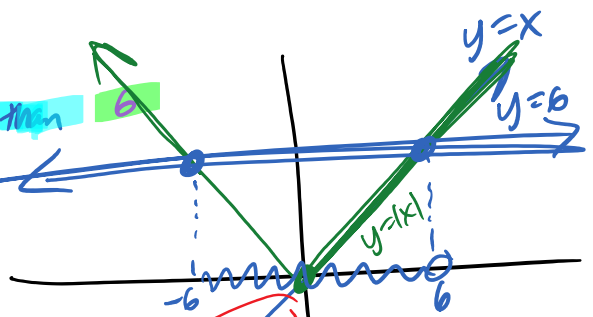
Ex: Solve $|x| < 6$. Verify the solutions graphically.

means dist b/t x & 0 on a # line is less than 6

$-6 < x < 6$

Interval notation: $(-6, 6)$

where is horiz line above ↑?



Ex: Solve $|2x-3| \leq 5$. Verify the solutions graphically.

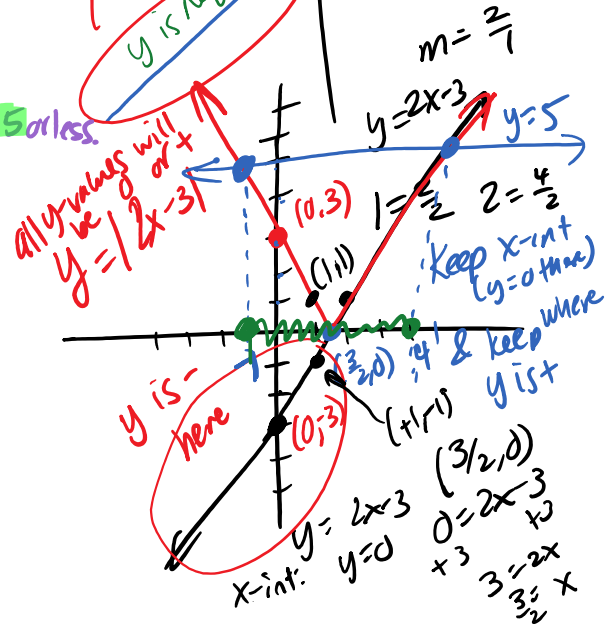
means dist b/t 0 & (2x−3) on a # line is 5 or less

$-5 \leq 2x-3 \leq 5$

$-\frac{2}{2} \leq \frac{2x}{2} \leq \frac{8}{2}$

$-1 \leq x \leq 4$

$[-1, 4]$



$|4x-1| > 6$

Ex: Solve $5|4x-1| > 30$. Verify the solutions graphically.

$|4x-1| > 6$ means dist b/t 0 & $4x-1$ on a # line is more than 6.

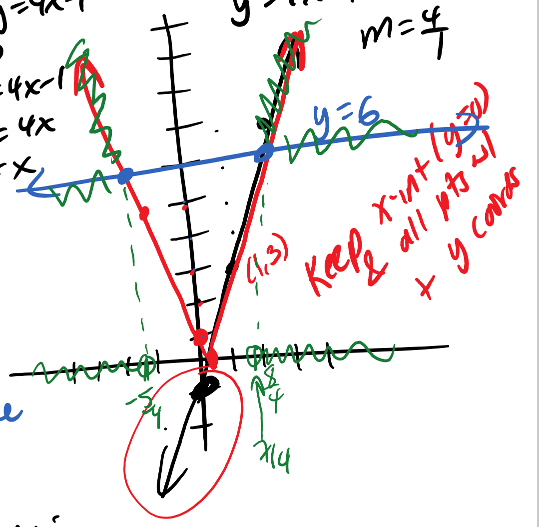
$4x-1 < -6$ or $4x-1 > 6$

$4x < -5$ or $4x > 7$

$x < -\frac{5}{4}$ or $x > \frac{7}{4}$

Interval notation: $(-\infty, -\frac{5}{4}) \cup (\frac{7}{4}, \infty)$

x-int of $y = 4x-1$: set $y=0$: $0 = 4x-1$, $1 = 4x$, $\frac{1}{4} = x$



Ex: Solve $3 < |2x-3| \le 5$. Verify the solutions graphically.

Solve $3 < |2x-3|$ AND $|2x-3| \le 5$

$|2x-3| > 3$ means dist b/t 0 & $(2x-3)$ on # line is more than 3

$2x-3 < -3$ or $3 < 2x-3$

$2x < 0$ or $2x > 6$

$x < 0$ or $x > 3$

$|2x-3| \le 5$ means dist b/t 0 & $(2x-3)$ on # line is 5 or less

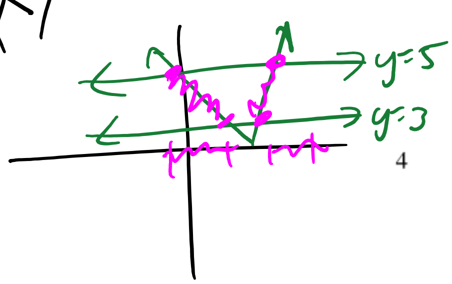
$-5 \le 2x-3 \le 5$

$-2 \le 2x \le 8$

$-1 \le x \le 4$

Put these together: Soln will satisfy BOTH

Interval notation: $[-1, 0) \cup (3, 4]$



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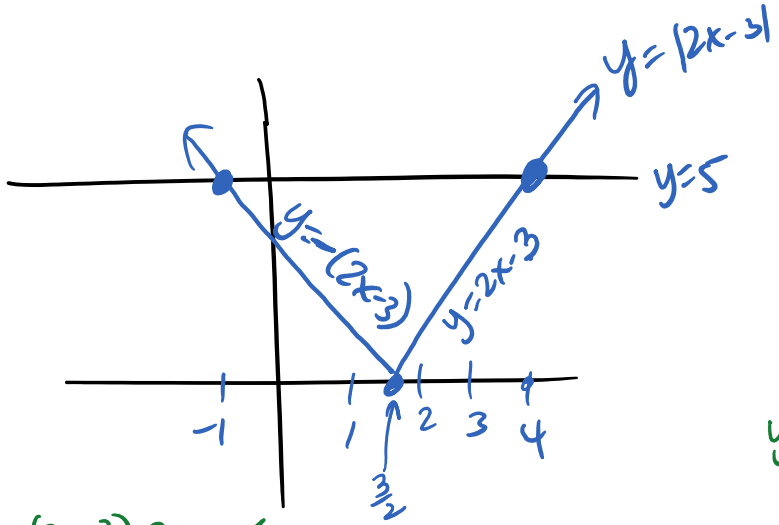


$y = -(2x-3)$ & $y = 5$

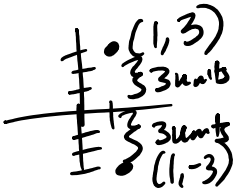
$-(2x-3) = 5$

$2x - 3 = -5$

$2x - 3 = 0$

$2x = 3$

$x = \frac{3}{2}$



$y = 2x - 3$ & $y = 5$

$2x - 3 = 5$