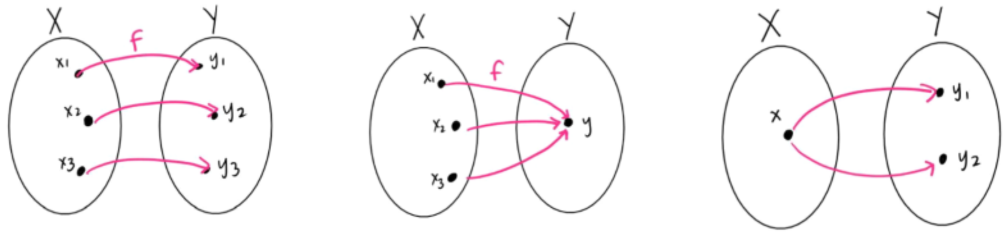
Math 1314 – College Algebra
Section 3.1 Functions and Function Notation

FUN FUN

- A relation is a set of points in a plane.
- A function is a correspondence that assigns one value of y to each value of x. (i.e. no repeated x-values)
- The set of input values (the set of all possible x-values) is the domain.
- The set of output values (the set of all possible y-values) is the range.

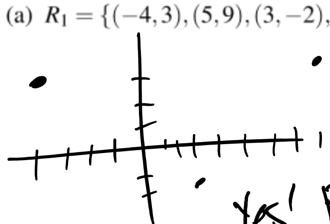


Not a fcn

- We use the Vertical Line Test to determine whether or not an expression is a function. If all vertical lines intersect the graph at most once, the expression is a function.

Ex: Function or not a function:

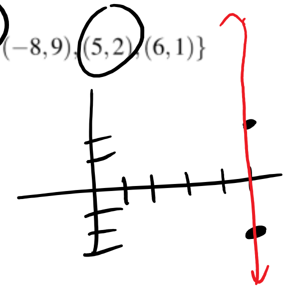
(a) $R_1 = \{(-4,3),(5,9),(3,-2),(7,3)\}$ ✓



Yes! Fcn!

(b) $R_2 = \{(5,-3),(-8,9),(5,2),(6,1)\}$

Not a fcn



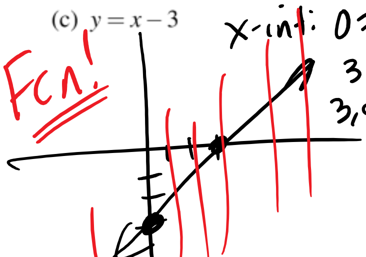
(c) $y = x - 3$

x-int: $0 = x - 3$

$3 = x$

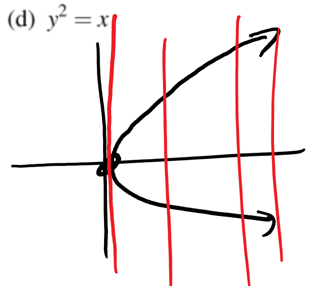
$(3,0)$

Fcn!

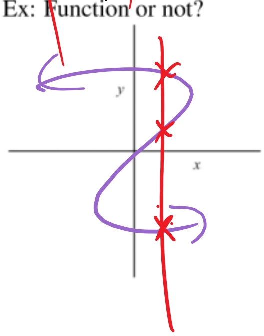


(d) $y^2 = x$

Not a fcn!



Ex: Function or not?



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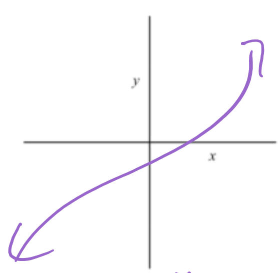
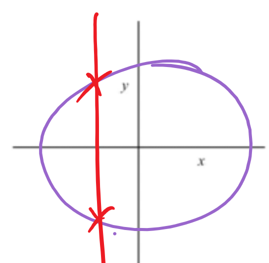
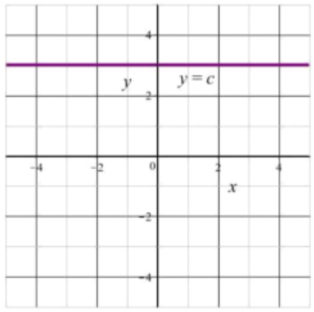


Fcn!!

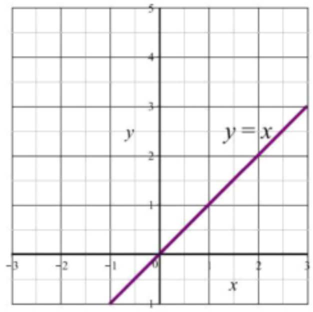


Not a fcn

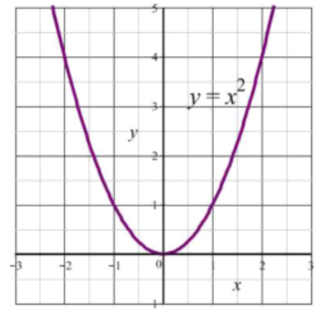
- Graphs of several basic functions:



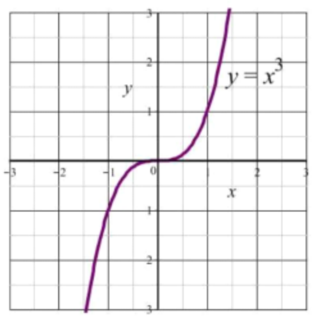
Constant Function



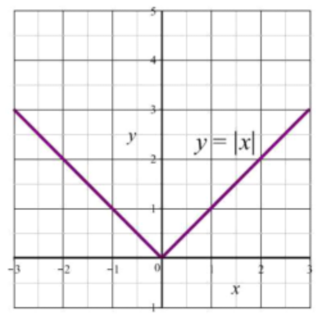
Identity Function



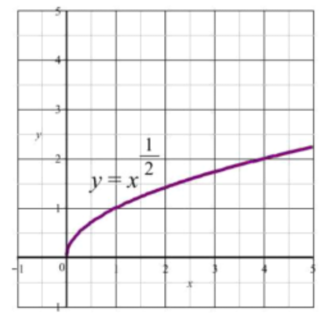
Squaring Function



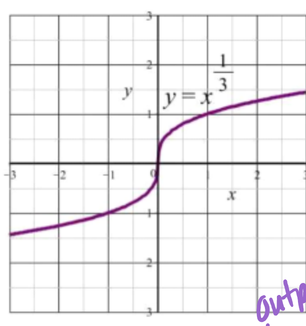
Cubing Function



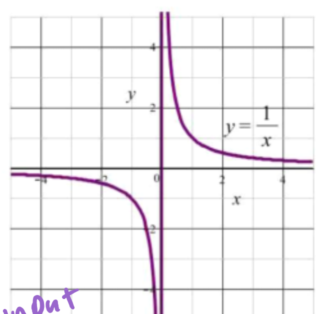
Absolute Value Function



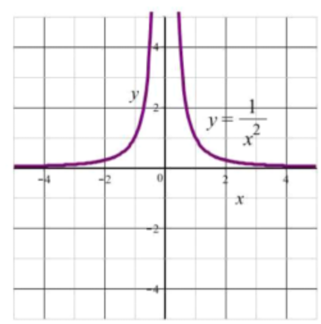
Square Root Function



Cube Root Function



Reciprocal Function



Reciprocal Squared Function

output input

$y = f(x)$ (x, y) or $(x, f(x))$

- Function notation: We write $y = f(x)$ to show that y is a function of x. We read $y = f(x)$ as y equals f of x.
- The independent variable is <u>x</u> and the dependent variable is <u>y</u>. y depends on x
- Think of the domain as the set of input values (the set of all possible x-values) and the range as the set of output values (the set of all possible y-values). The function f tells us what to do to the input value. Think of f as a process or operation.

set y=0

Set $y=0$

$x^2+4x=0$

$x(x+4)=0$

$x=0 \quad x=-4$

$(0,0) \quad (-4,0)$

- For example, the function $f(x) = x^2 + 4x$ takes the input value and squares it. Then it adds that new value to 4 times the input value.

Ex: Let $f(x) = x^2 + 4x$.

$y = x^2 + 4x$
Find y when x is -1.

(a) Find $f(-1)$. $= (-1)^2 + 4(-1)$
$= 1 + -4 = -3$

input is $x=-1$. output is $y=-3$
can write a point: $(-1,-3)$ is a point on the graph of $f(x) = x^2 + 4x$

(b) Find $f(5)$. $= 5^2 + 4(5)$
$= 25 + 20 = 45$

point: $(5,45)$

(c) Find $f(3)$. $= 3^2 + 4(3)$
$= 9 + 12 = 21$

point: $(3,21)$

$f(x) = x^2 + 4x$

(d) Find $f(\star)$. $= \star^2 + 4\star$

$(a+b)^2 = a^2 + 2ab + b^2$
sq front, 2(front)(back), back sq.

(e) Find $f(x+h)$. $= (x+h)^2 + 4(x+h)$
$= x^2 + 2xh + h^2 + 4x + 4h$

$f(x) = x^2 + 4x$

(f) Find $\dfrac{f(x+h) - f(x)}{h}$ $= \dfrac{x^2 + 2xh + h^2 + 4x + 4h - \overbrace{(x^2 + 4x)}^{f(x)}}{h}$ Difference Quotient

$= \dfrac{\cancel{x^2} + 2xh + h^2 + \cancel{4x} + 4h - \cancel{x^2} - \cancel{4x}}{h}$

every term that does not have an h in it in num. will cancel or you messed up!

$= \dfrac{2xh + h^2 + 4h}{h} = \dfrac{\cancel{h}(2x+h+4)}{\cancel{h}} = \boxed{2x+h+4}$

$f(x) = x^2 + 4x$

(g) Solve $f(x) = 5$. ← given output

$5 = x^2 + 4x$

$-1 \quad +5$

$0 = x^2 + 4x - 5$

$0 = (x-1)(x+5)$

$x - 1 = 0$, $+1 \; +1$, $x = 1$ $\qquad$ $x + 5 = 0$, $-5 \; -5$, $x = -5$

$x = 1$ or $x = -5$

Ex: Given $f(x) = \sqrt{5-x}$, find

(a) $f(-6) = \sqrt{5-(-6)} = \sqrt{5+6} = \sqrt{11}$

(b) $f(100) = \sqrt{5-100} = \sqrt{-95}$ ✗ Imaginary

x=100 is not in the domain

$f(x) = \sqrt{5-x}$

(c) $f(m+3) = \sqrt{5-(m+3)}$

$= \sqrt{5-m-3}$

$= \sqrt{2-m}$

$f(m+3) \neq f(m) + f(3)$

not equal

(d) $f(m) + f(3)$

$= \sqrt{5-m} + \sqrt{5-3}$

$= \sqrt{5-m} + \sqrt{2}$

(e) $\frac{f(x)}{5} = \frac{\sqrt{5-x}}{5}$

$f(x) = \sqrt{5-x}$

(f) Solve $f(x) = 5$

$\sqrt{5-x} = 5$

Sq both sides:

$5-x = 25$

$-x = 20$

$\boxed{x = -20}$

check: $\sqrt{5-(-20)} \stackrel{?}{=} 5$

$\sqrt{5+20} \stackrel{?}{=} 5$

$\sqrt{25} = 5$ ✓

Ex: Given $f(x) = \frac{x-2}{x+3}$, find

(a) $f(4) = \frac{4-2}{4+3} = \frac{2}{7}$

(b) $f(k-2) = \frac{(k-2)-2}{(k-2)+3} = \frac{k-2-2}{k-2+3} = \boxed{\frac{k-4}{k+1}}$

not equal

(c) $f(k) - f(2) = \frac{k-2}{k+3} - \frac{2-2}{2+3}$
$= \frac{k-2}{k+3} - \frac{0}{5} = \boxed{\frac{k-2}{k+3}}$

(d) $f(a+h)$
$= \frac{(a+h)-2}{(a+h)+3} = \boxed{\frac{a+h-2}{a+h+3}}$

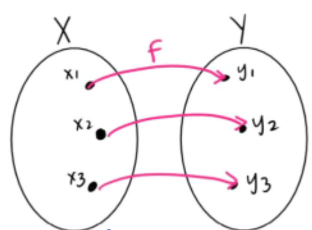
$\frac{3\cancel{7}5}{2\cancel{7}1} = \frac{35}{21}$

$\frac{300+\cancel{70}+5}{300+\cancel{70}+1}$

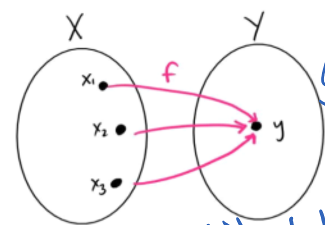
$\frac{0}{5} = 0$

$f(x) = \frac{x-2}{x+3}$

- A function f from a set A to a set B is one-to-one if and only if different numbers in the domain of f have different outputs in the range of f.
- 1-1: If x_1 and x_2 are in the domain of f and $x_1 \neq x_2$, then $f(x_1) \neq f(x_2)$



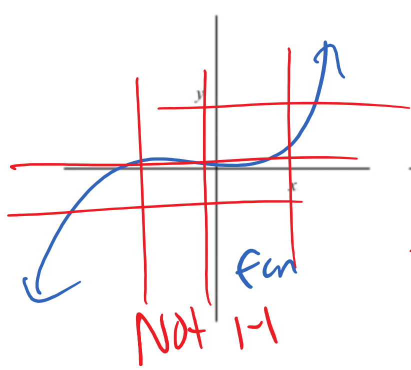
fcn
also 1-1



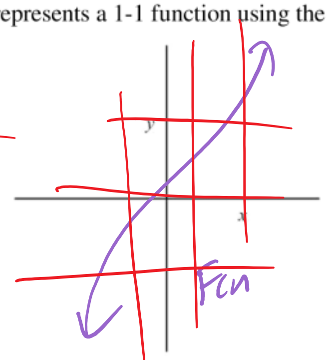
y here is not unique

fcn Not 1-1

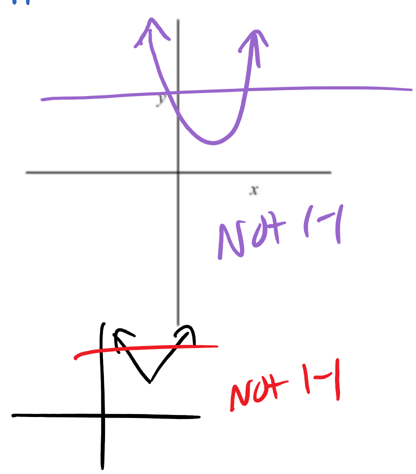
- We can test to determine if a graph represents a 1-1 function using the Horizontal Line Test.



fcn
Not 1-1



fcn
1-1 fcn



Not 1-1

Not 1-1