Math 1314 – College Algebra
Section 3.2 Domain and Range

- Recall: Think of the domain as the set of input values (the set of all possible x-values) and the range as the set of output values (the set of all possible y-values). The function f tells us what to do to the input value. Think of f as a process or operation.

Summary for Domain of Polynomial, Radical, and Rational Functions:	
▪Look for fractions with variables in the denominator. Solve denominator $\neq 0$.	
▪Look for even roots with variables in the radicand. Solve radicand ≥ 0.	
Form:	To find domain:
$f(x) = \dfrac{\text{anything}}{x \text{ stuff}}$	Solve x stuff $\neq 0$
$f(x) = \sqrt[\text{even}]{x \text{ stuff}}$	Solve x stuff ≥ 0

Recall: The radicand is the part of the function that is under the root.

Ex: Find the domain of the function

denom has variables!
denom $\neq 0$

if $ab=0$, then $a=0$ or $b=0$

(a) $y = \dfrac{4x+3}{x^2-x-6}$

$+1 \quad -6$
$+2 \quad -3$

$y = \dfrac{4x+3}{(x+2)(x-3)}$

$(x+2)(x-3) \neq 0$
$x+2 \neq 0$ AND $x-3 \neq 0$
$x \neq -2$ AND $x \neq 3$

if $ab \neq 0$, then $a \neq 0$ AND $b \neq 0$

$-2 \quad 3$

Domain: $(-\infty,-2) \cup (-2,3) \cup (3,\infty)$

(b) $f(x) = 6x+1$

Domain: $(-\infty,\infty)$
$\mathbb{R}$

(c) $g(x) = \sqrt{2-3x}$

even root. check radicand: it has variables!
radicand ≥ 0

$2-3x \geq 0$
$+3x \quad +3x$
$\dfrac{2}{3} \geq \dfrac{3x}{3}$
$\dfrac{2}{3} \geq x$
$x \leq \dfrac{2}{3}$

$\frac{2}{3}$

Domain: $(-\infty, \frac{2}{3}]$

$g(0) = \sqrt{2-0} = \sqrt{2}$
$g(100) = \sqrt{2-300}$
$\sqrt{-298}$ ✗

If only thing in denom is sqrt: denom $\neq 0$ AND radicand ≥ 0 $\Rightarrow$ radicand > 0

(d) $y = -\dfrac{1}{\sqrt{x+3}}$

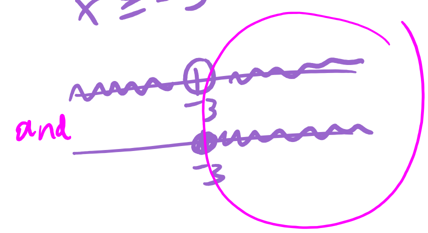
denom $\neq 0$ and radicand ≥ 0

$\sqrt{x+3} \neq 0$ and $x+3 \geq 0$

Sq. both sides: $x \geq -3$

$x+3 \neq 0$

$x \neq -3$



Domain: $(-3, \infty)$

(e) $h(x) = \sqrt[3]{x+2}$ index here is odd: cube root

Domain: $\mathbb{R}$ or $(-\infty, \infty)$

(f) $j(x) = \dfrac{\sqrt{5-x}}{x^2-16}$

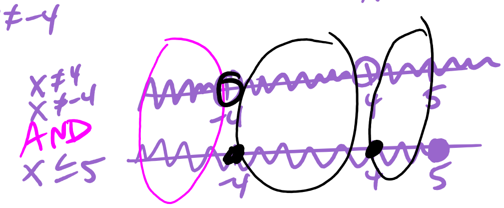
denom $\neq 0$ even root: radicand ≥ 0

$x^2 - 16 \neq 0$ $5 - x \geq 0$

$x^2 \neq 16$ AND $5 \geq x$

$\sqrt{x^2} \neq \pm\sqrt{16}$ $x \neq 4$ and $x \leq 5$

$x \neq -4$



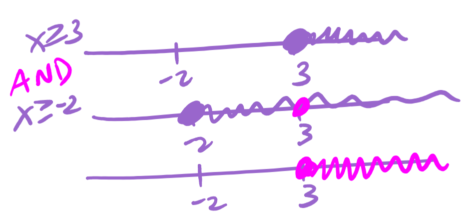
Domain: $(-\infty, -4) \cup (-4, 4) \cup (4, 5]$

(g) $m(x) = \sqrt{x-3} + \sqrt{2+x}$

radicand ≥ 0 and radicand ≥ 0

$x - 3 \geq 0$ and $2 + x \geq 0$

$x \geq 3$ and $x \geq -2$



Domain: $[3, \infty)$

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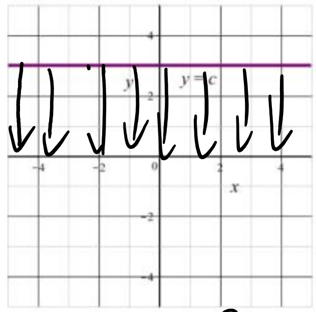
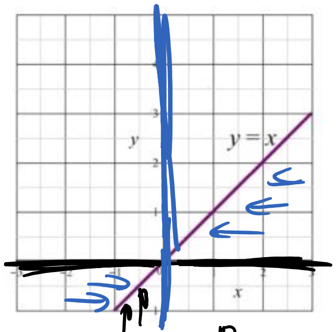
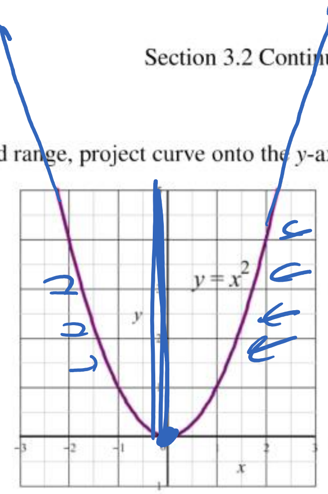
- We can find the domain and range of functions using graphs.
  - To find domain, project curve onto the x-axis.
  - To find range, project curve onto the y-axis.



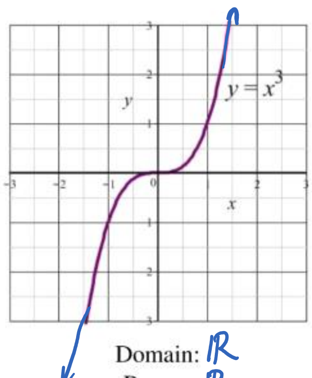
Domain: $\mathbb{R}$
Range: $\{c\}$
Constant Function



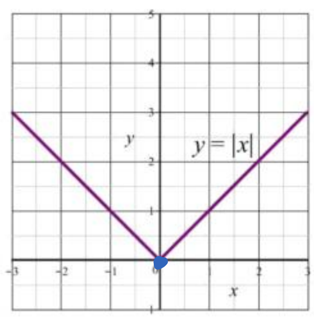
Domain: $\mathbb{R}$
Range: $(-\infty,\infty)$ or $\mathbb{R}$
Identity Function



Domain: $\mathbb{R}$
Range: $[0,\infty)$
Squaring Function

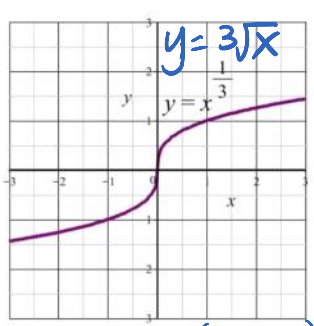


Domain: $\mathbb{R}$
Range: $\mathbb{R}$
Cubing Function

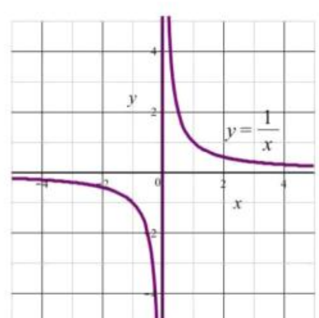


Domain: $\mathbb{R}$
Range: $[0,\infty)$
Absolute Value Function

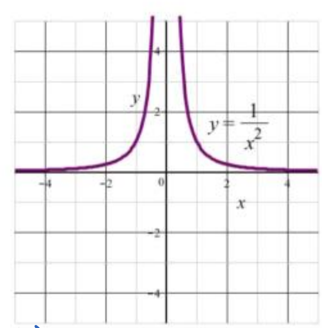
Domain: $[0,\infty)$
Range: $[0,\infty)$
Square Root Function



Domain: $(-\infty,\infty)$ or $\mathbb{R}$
Range: $(-\infty,\infty)$ or $\mathbb{R}$
Cube Root Function



Domain: $(-\infty,0)\cup(0,\infty)$
Range: $(-\infty,0)\cup(0,\infty)$
Reciprocal Function

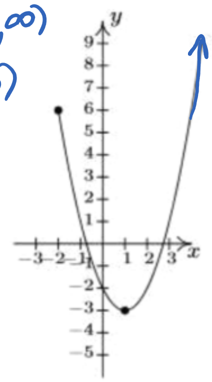


Domain: $(-\infty,0)\cup(0,\infty)$
Range: $(0,\infty)$
Reciprocal Squared Function

Ex: Find the domain and range of the function from the graph.

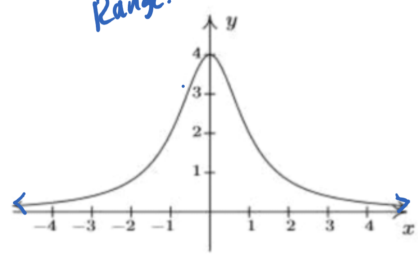
(a)

Domain: $[-2, \infty)$
Range: $[-3, \infty)$



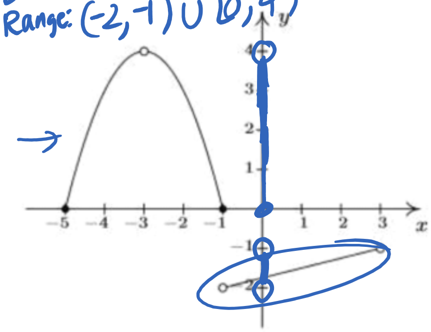
(b)

Domain: $\mathbb{R}$
Range: $(0, 4]$

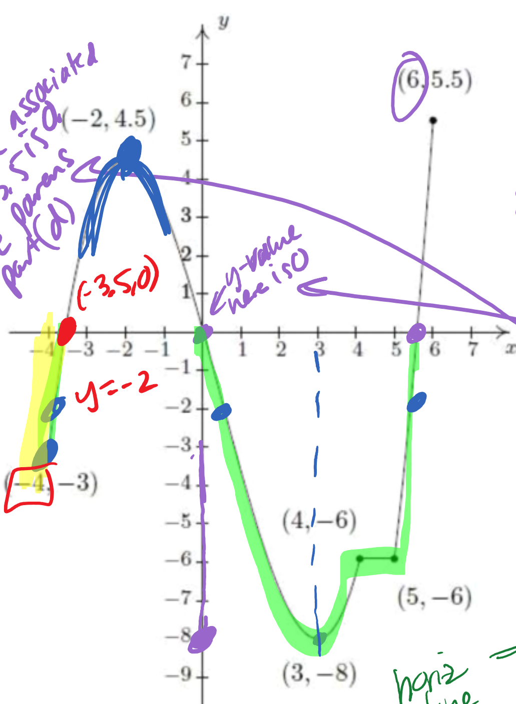


(c)

Domain: $[-5, -3) \cup (-3, 3)$
Range: $(-2, -1) \cup [0, 4)$



Ex: Given the graph of $y = f(x)$ below, answer the following questions.



y-value associated w/ $x = -3.5$ is 0 so use parens for part (d)

(−3.5, 0)

$y = -2$

y-value here is 0

horiz line

Interval of x-values

Interval of y-values

(a) Find the domain of f. Find the range of f.

Domain: $[-4, 6]$ Range: $[-8, 5.5]$

(b) Find the x-intercepts and y-intercept. Write as points.

x-int: $(-3.5, 0), (0, 0), (5.5, 0)$ y-int: $(0, 0)$

(c) Find the zeroes of f. x-values for which y-value is 0

Zeros: $x = -3.5$, $x = 0$, $x = 5.5$

(d) Find $f(x) < 0$ where y is negative

$[-4, -3.5) \cup (0, 5.5)$

(e) Find $f(3) = -8$

look at $y = -2$

(f) Find the number of solutions to $f(x) = -2$.

3 solns

(g) Find the intervals where f is increasing, constant, and decreasing.

Increasing: $(-4, -2) \cup (3, 4) \cup (5, 6)$
constant: $(4, 5)$ decr: $(-2, 3)$

(h) Find the local minimums and local ~~minimums~~ maximums, if any exist. "2 sides"

local max at $(-2, 4.5)$. local min at $(3, -8)$
local max is 4.5 local min of −8.

(i) Find the maximum and minimum, if it exists.

Max of 5.5 at $(6, 5.5)$
min of −8 at $(3, -8)$

for (d): $f(x) < 0$
use parentheses when the x-coord. has a y-value of 0. we need that value b/c it's the edge of the interval, but we can't include it b/c 0 is not −

Piecewise fcn

Ex: Find the domain and graph:

(a) $f(x) = \begin{cases} x^2 + 2x & \text{if } x \le -1 \\ 3 - x & \text{if } x > -1 \end{cases}$

① $y = x^2 + 2x$

x-int: Set $y=0$

$0 = x^2 + 2x$

$0 = x(x+2)$

$x = 0$ or $x + 2 = 0$

$x = -2$

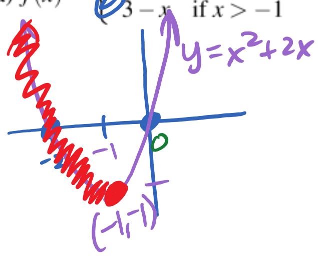
$(0,0)$ $(-2,0)$

when $x=-1$, find y.

$x=-1$: $y = (-1)^2 + 2(-1)$

$y = 1 - 2 = -1$

new point: $(-1,-1)$



② $y = 3 - x = -x + 3$ (mx+b)

$m = -1$ y-int $(0,3)$

$y = 3 - x$

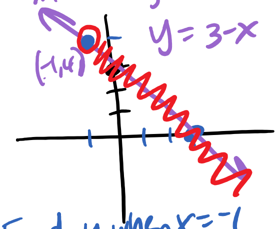
x-int: set $y=0$:

$0 = 3 - x$

$+x$ $+x$

$x = 3$

$(3,0)$

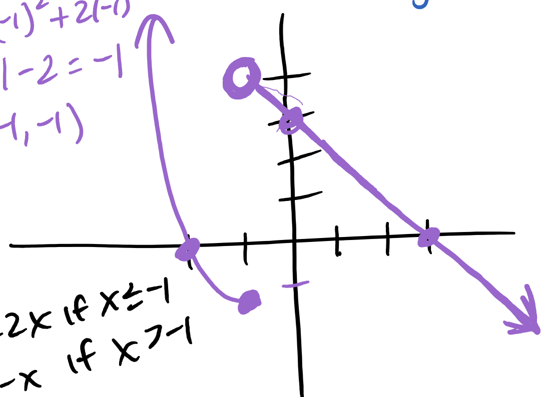


Find y when $x=-1$

$y = 3 - (-1)$

$y = 3 + 1 = 4$

pt $(-1,4)$



$f(x) = \begin{cases} x^2 + 2x & \text{if } x \le -1 \\ 3 - x & \text{if } x > -1 \end{cases}$

Find $f(-2)$.

x-value.

here, $x=-2$

check $-2 \le -1$ ⇒ use parabola piece.

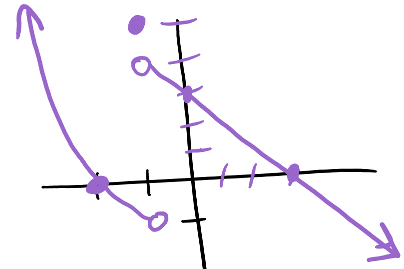
$-2 > -1$

Plug $x=-2$ into top:

$f(-2) = (-2)^2 + 2(-2)$

$= 4 - 4 = 0$

(b) $g(x) = \begin{cases} x^2 + 2x & \text{if } x < -1 \\ 5 & \text{if } x = -1 \\ 3 - x & \text{if } x > -1 \end{cases}$ open circle $(-1,-1)$, pt $(-1,5)$



$-5 < -1$ use parabola

$g(-5) = (-5)^2 + 2(-5) = 25 - 10 = 15$

$-1 = -1$ use middle piece

$g(-1) = 5$

$11 > -1$ use line

$g(11) = 3 - 11 = -8$

$-1 \le -1$ use parabola

$f(-1) = (-1)^2 + 2(-1) = 1 - 2 = -1$

$7 > -1$ use line

$f(7) = 3 - 7 = -4$

(c) Find $f(-2), f(-1), f(7), g(-5), g(-1)$ and $g(11)$

- Practice lots of graphing BY HAND!!