Math 1314 – College Algebra
Section 3.4 Function Composition

Function Arithmetic:
The domain of each function, unless otherwise restricted, is the set of real numbers x that are in the domains of f and g.
- $(f+g)(x) = f(x)+g(x)$
- $(f-g)(x) = f(x)-g(x)$
- $(fg)(x) = f(x)g(x)$
- $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, g(x) \neq 0$

Ex: Suppose $f(x) = 3x-1$ and $g(x) = 2x-6$. Find each function and its domain.

(a) $(f+g)(x) = f(x) + g(x)$
$= 3x-1 + 2x-6$
$(f+g)(x) = 5x - 7$
Domain: $\mathbb{R}$

(b) $(fg)(x) = f(x)g(x)$
$= (3x-1)(2x-6)$
$= 6x^2 - 18x - 2x + 6$
$= 6x^2 - 20x + 6$
Domain: $\mathbb{R}$

Ex: Let $f(x) = \sqrt{x+2}$ and $g(x) = x^2+5$. Find $\left(\frac{g}{f}\right)(x)$ and its domain.

$\left(\frac{g}{f}\right)(x) = \frac{g(x)}{f(x)} = \frac{x^2+5}{\sqrt{x+2}}$

Domain: denom $\neq 0$ AND radicand ≥ 0
$\sqrt{x+2} \neq 0$ $x+2 \geq 0$
$x+2 \neq 0$ $x \geq -2$
$x \neq -2$
-2 -2
Sol: -2

Domain: $(-2, \infty)$

Ex: If $f(x) = \frac{x+3}{x-2}$ and $g(x) = \frac{2}{x}$, find $(f-g)(x)$ and its domain.

$(f-g)(x) = f(x) - g(x)$
$\boxed{(f-g)(x) = \frac{x+3}{x-2} - \frac{2}{x}}$

Domain: denom $\neq 0$ and denom $\neq 0$
$x-2 \neq 0$ $x \neq 0$
$x \neq 2$
0 2

Domain: $(-\infty, 0) \cup (0, 2) \cup (2, \infty)$

frac/frac: ① = ② rewrite frac in num ③ multiply by recip of frac in denom.

$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{\frac{x+3}{x-2}}{\left(\frac{2}{x}\right)} = \frac{(x+3)}{(x-2)} \cdot \frac{(x)}{(2)} = \frac{x(x+3)}{2(x-2)}$

Domain: denom $\neq 0$ $2(x-2) \neq 0$ $x \neq 2$ and $x \neq 0$
$(-\infty, 0) \cup (2, 0) \cup (2, \infty)$

Ex: Let $f(x) = \sqrt{x+1}$ and $g(x) = x-4$. Find $(f-g)(11)$

$(f-g)(11) = f(11) - g(11)$
$= \sqrt{11+1} - (11-4)$
$= \sqrt{12} - 7$
$= 2\sqrt{3} - 7$

12 → ② 6 → ② ③
$\sqrt{12} = \sqrt{2 \cdot 2 \cdot 3}$
$= 2\sqrt{3}$

- Composition of functions: $(f \circ g)(x) = \underbrace{f(g(x))}_{f \text{ of } g \text{ of } x}$

f composed w/ g w/ an input of x

Domain:
① Find domain of inner fcn
② Simplify the composition & find the domain of that fcn.
③ Take intersection of ① & ②

$f(56)=\frac{56+3}{56-2}$ $f(103)=\frac{103+3}{103-2}$ $f(19.1)=\frac{19.1+3}{19.1-2}$ $f(8)=\frac{8+3}{8-2}$

$f(87)=\frac{87+3}{87-2}$ $f(15)=\frac{15+3}{15-2}$

Ex: If $f(x)=\dfrac{x+3}{x-2}$ and $g(x)=\dfrac{2}{x}$, find

(a) $f(5)=\frac{5+3}{5-2}$ (b) $f(-3)=\frac{-3+3}{-3-2}$ (c) $f(0)=\frac{0+3}{0-2}$ (d) $f(\star)=\frac{\star+3}{\star-2}$

(e) $(f\circ g)(x)$ and its domain.

$$(f\circ g)(x)=f(g(x))=f\left(\underset{①}{\frac{2}{x}}\right)=\frac{\frac{2}{x}+3}{\frac{2}{x}-2}=\frac{\frac{2}{x}+\frac{3\cdot x}{1\cdot x}}{\frac{2}{x}-\frac{2\cdot x}{1\cdot x}}=\frac{\frac{2}{x}+\frac{3x}{x}}{\frac{2}{x}-\frac{2x}{x}}$$

$$=\frac{\left(\frac{2+3x}{x}\right)}{\left(\frac{2-2x}{x}\right)}=\left(\frac{2+3x}{\cancel{x}}\right)\left(\frac{\cancel{x}}{2-2x}\right)=\boxed{\frac{2+3x}{2-2x}}\ ②$$

① ② rewrite frac w/ num ③ mult. by recip of frac in denom.

0 1

Domain: ① inner fn $\frac{2}{x}$: $x\neq 0$
② simplified ans: $\frac{2+3x}{2-2x}$ $2-2x\neq 0$, $2\neq 2x$, $1\neq x$

Domain: $(-\infty,0)\cup(0,1)\cup(1,\infty)$

(f) $(g\circ g)(x)$ and its domain.

$f(x)=\frac{x+3}{x-2}$ $g(x)=\frac{2}{x}$

$$=g(g(x))=g\left(\underset{1}{\frac{2}{x}}\right)=\frac{2}{\left(\frac{2}{x}\right)}=\left(\frac{\cancel{2}}{1}\right)\left(\frac{x}{\cancel{2}}\right)=x\ \boxed{2}$$

① ② rewrite num ③ mult by recip of denom

frac / frac

So $(g\circ g)(x)=x$

$g(5)=\frac{2}{5}$ $g(6)=\frac{2}{6}$ $g(11)=\frac{2}{11}$ $g(-3)=\frac{2}{-3}$ $g(\pi)=\frac{2}{\pi}$ $g(19)=\frac{2}{19}$ $g\left(\frac{2}{x}\right)=\frac{2}{\frac{2}{x}}$

Domain: $\boxed{1}$ inner fn: $\frac{2}{x}$: $x\neq 0$ $\Rightarrow x\neq 0$
$\boxed{2}$ final simplified answer: x: $\mathbb{R}$

0

Domain: $(-\infty,0)\cup(0,\infty)$

Ex: Let f be the function defined by $f=\{(-3,1),(-2,2),(-1,1),(0,9),(1,8),(2,9),(3,-3)\}$ and let g be the function defined $g=\{(-3,-4),(-2,3),(-1,-6),(0,8),(1,-6),(2,5),(3,2)\}$.
Find $(f\circ g)(-2)$ if it exists.

$(f\circ g)(-2)=f(\underbrace{g(-2)})=f(3)=\boxed{-3}$

$(f\circ g)(3)=f(g(3))=f(2)=\boxed{9}$

Ex: If a rack of winter clothing is marked 30% off and you have a coupon for an additional 20% off, find a price function $P(x)$ giving the final price you pay on an item that originally costs \$$x$.

$f(x)=.70\boxed{x}$
$g(x)=.80x$

30% discount ⇒ pay 100% − 30% = 70%
20% discount ⇒ pay 80%

$f(2)=.7(2)$
$f(9)=.7(9)$
$f(-5)=.7(-5)$
$f(100)=.7(100)$
$f(83)=.7(83)$
$f(.80x)=.7(.80x)$

$P(x)=(f\circ g)(x)=f(g(x))=f(.80x)=.7(.80x)=.56x$

Ex: Let $f(x) = \sqrt{x+1}$ and $g(x) = x-4$. Find

(a) $f(7) = \sqrt{7+1} = \sqrt{8} = 2\sqrt{2}$

(b) $f(11) = \sqrt{11+1} = \sqrt{12} = 2\sqrt{3}$

(c) $f(-1) = \sqrt{-1+1} = 0$

(d) $f(\star) = \sqrt{\star+1}$

$f(5) = \sqrt{5+1}$ $\quad f(37) = \sqrt{37+1}$ $\quad f(106) = \sqrt{106+1}$

$f(84) = \sqrt{84+1}$ $\quad f(11345) = \sqrt{11345+1}$

$f(2.3729) = \sqrt{2.3729+1}$

(e) $(f \circ g)(x)$ and its domain

$= f(g(x)) = f(x-4) = \sqrt{x-4+1}$

$= \boxed{\sqrt{x-3}}$

Domain: ① inner fcn: $x-4$: $\mathbb{R}$

② final simplified ans $\sqrt{x-3}$: $x-3 \geq 0$, $x \geq 3$

Domain of $(f \circ g)(x) = [3, \infty)$

(f) $(g \circ f)(x)$ and its domain

$f(x) = \sqrt{x+1}$ $\quad g(x) = x-4$

$= g(f(x)) = g(\sqrt{x+1}) = \boxed{\sqrt{x+1} - 4}$

$g(5) = 5-4$
$g(13) = 13-4$
$g(186) = 186-4$
$g(92) = 92-4$
$g(-13) = -13-4$
$g(51) = 51-4$
$g(\sqrt{x+1}) = \sqrt{x+1} - 4$

Domain: ① inner fcn: $\sqrt{x+1}$: radicand ≥ 0, $x+1 \geq 0$, $x \geq -1$

② final simplified ans: $\sqrt{x+1} - 4$, radicand ≥ 0, $x+1 \geq 0$

Domain: $[-1, \infty)$

Ex: Find functions f and g such that $h = f \circ g$.

(a) $h(x) = \sqrt[3]{x^2 - 2x + 5}$

inner fcn: $x^2 - 2x + 5 = g(x)$

outer fcn: $f(x) = \sqrt[3]{x}$

$f(g(x)) = f(x^2 - 2x + 5)$

or $f(x) = \sqrt[3]{x+5}$, $g(x) = x^2 - 2x$

(b) $h(x) = \dfrac{1}{(x-3)^5} + (x-3)^5$

inner: $g(x) = (x-3)^5$

outer: $f(x) = \frac{1}{x} + x$

or $f(x) = \frac{1}{x^5} + x^5$, $g(x) = x-3$

Ex: The number of cars running in the business district of a town at time t is given by $f(t)$. Carbon monoxide pollution coming from these cars is given by $g(x)$ parts per million, where x is the number of cars being operated in the district. What does the function $(g \circ f)(t)$ represent?

$f(t) =$ # of cars in terms of time

$g(x) =$ CO pollution in terms of # of cars

$x =$ # of cars

$(g \circ f)(t) = g(f(t)) = g(x)$ input changed to x

$t =$ time; x: # of cars

$\boxed{(g \circ f)(t) \text{ gives CO pollution as a fcn of time}}$