

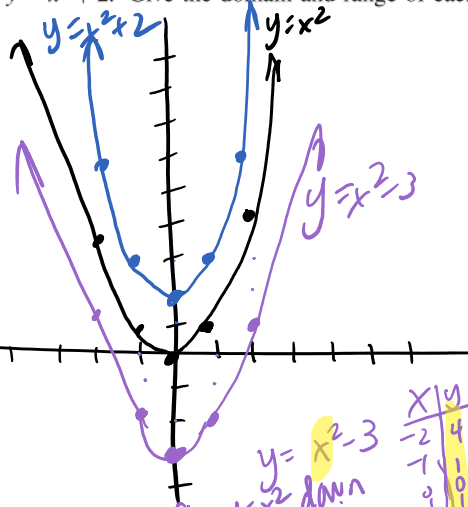
## Math 1314 – College Algebra Section 3.5 Transformation of Functions

Ex: Graph  $y = x^2$  and  $y = x^2 + 2$ . Give the domain and range of each. Also identify where each function is increasing or decreasing.

$y = x^2$

x	y
-2	4
-1	1
0	0
1	1
2	4

$y = x^2$   
 Domain:  $\mathbb{R}$   
 Range:  $[0, \infty)$   
 incr:  $(0, \infty)$   
 decr:  $(-\infty, 0)$   
 local min at  $(0, 0)$



$y = x^2 + 2$

x	y
-2	4+2=6
-1	1+2=3
0	0+2=2
1	1+2=3
2	4+2=6

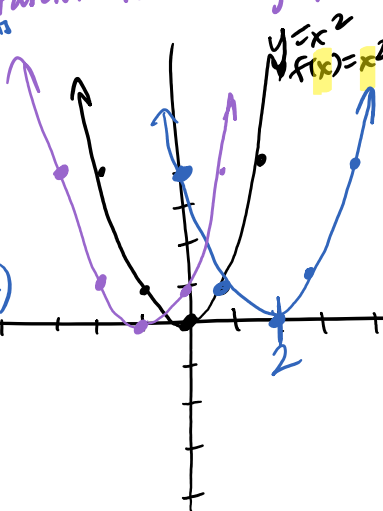
Shift  $y = x^2$  up 2 units  
 $y = x^2 + 2$   
 Domain:  $\mathbb{R}$   
 Range:  $[2, \infty)$   
 local min at  $(0, 2)$   
 incr:  $(0, \infty)$   
 decr:  $(-\infty, 0)$

Ex: Graph  $y = (x-2)^2$  and  $y = (x+1)^2$ . Give the domain and range of each. Also identify where each function is increasing or decreasing.

$f(x) = x^2$   
 $y = x^2$

x	y
-2	4
-1	1
0	0
1	1
2	4

$y = (x-2)^2$   
 Shift  $y = x^2$  right 2 units  
 Domain:  $\mathbb{R}$   
 Range:  $[0, \infty)$   
 incr:  $(2, \infty)$   
 decr:  $(-\infty, 2)$



$y = (x+1)^2$  replaced  $x$  w/  $x+1$   
 Shift  $y = x^2$  left 1 unit

x	y
-3	4
-2	1
-1	0
0	1
1	4

Domain:  $\mathbb{R}$   
 Range:  $[0, \infty)$   
 incr:  $(-1, \infty)$   
 decr:  $(-\infty, -1)$

Vertical and Horizontal Shifts: Suppose  $k > 0$ .

- (a) To graph  $y = f(x) + k$ : Shift graph of  $y = f(x)$   $k$  units upward
- (b) To graph  $y = f(x) - k$ : Shift graph of  $y = f(x)$   $k$  units downward
- (c) To graph  $y = f(x + k)$ : Shift graph of  $y = f(x)$   $k$  units left
- (d) To graph  $y = f(x - k)$ : Shift graph of  $y = f(x)$   $k$  units right

Ex:  $y = (x-2)^2$   
 $y = x^2$  for  $(0, 0)$   
 $0 = (x-2)^2$   
 $0 = x-2$   
 $x = 2$

NOTE: When  $x$  has been replaced with another function, like  $x + \#$  or  $x - \#$  (think: composition of functions), we are looking at a horizontal shift.

Ex:  $y = (x+1)^2$  shifts graph of  $y = x^2$  LEFT 1 unit.

Consider  $y = x^2$ : turning pt is  $(0, 0)$   
 (where  $y$ -value is 0)

Where will  $y=0$  in new fn? new pt where  $y=0$  is  $(-1, 0)$   
 $0 = (x+1)^2$   
 $0 = x+1$   
 $-1 = x$   
 moved turning pt LEFT 1 unit

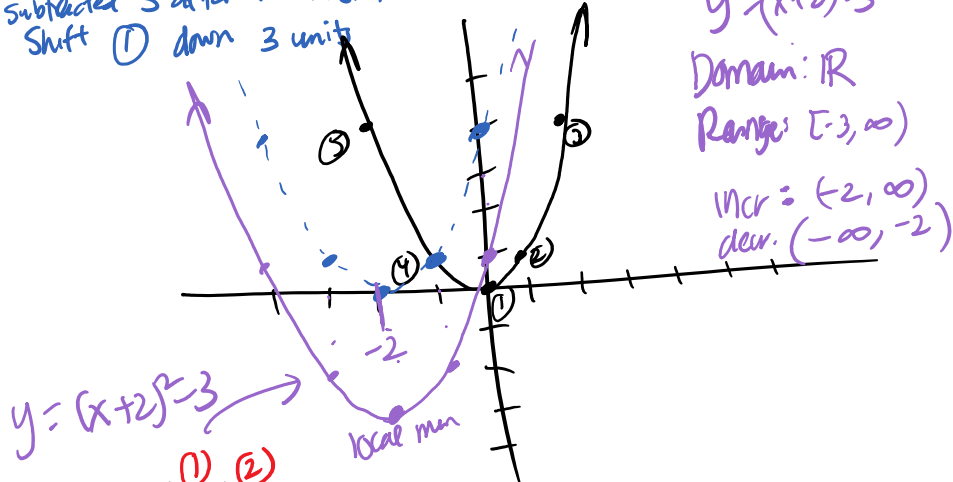
Ex: Graph  $y = (x+2)^2 - 3$ . Find the domain and range.

Parent fcn:  $y = x^2$

① replaced  $x$  w/  $x+2$   $\Rightarrow$  comparison of fcn  $\Rightarrow$  horizontal shift: solve  $x+2=0$   
 $x = -2$   
 Shift  $y = x^2$  LEFT 2 units

② subtracted 3 after the  $x$  shift  $\Rightarrow$  vertical shift  
 Shift ① down 3 units

x	y
-2	4
-1	1
0	0
1	1
2	4



$y = (x+2)^2 - 3$   
 Domain:  $\mathbb{R}$   
 Range:  $[-3, \infty)$   
 Incr:  $(-2, \infty)$   
 Decr:  $(-\infty, -2)$

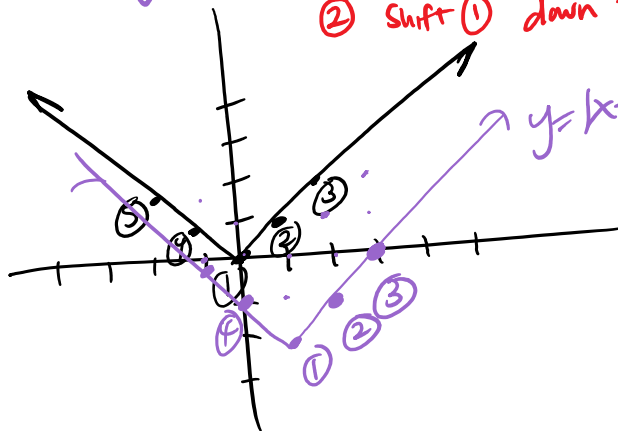
Ex: Graph  $y = |x-1| - 2$ . Find the domain and range.

Parent fcn:  $y = |x|$

① shift  $y = |x|$  right 1  
 ② shift ① down 2

$y = |x|$

x	y
-2	2
-1	1
0	0
1	1
2	2



Domain:  $\mathbb{R}$   
 Range:  $[-2, \infty)$

Ex: Graph  $f(x) = \sqrt{x}$ ,  $g(x) = \sqrt{-x}$ , and  $h(x) = -\sqrt{x}$ . Find the domain and range of each.

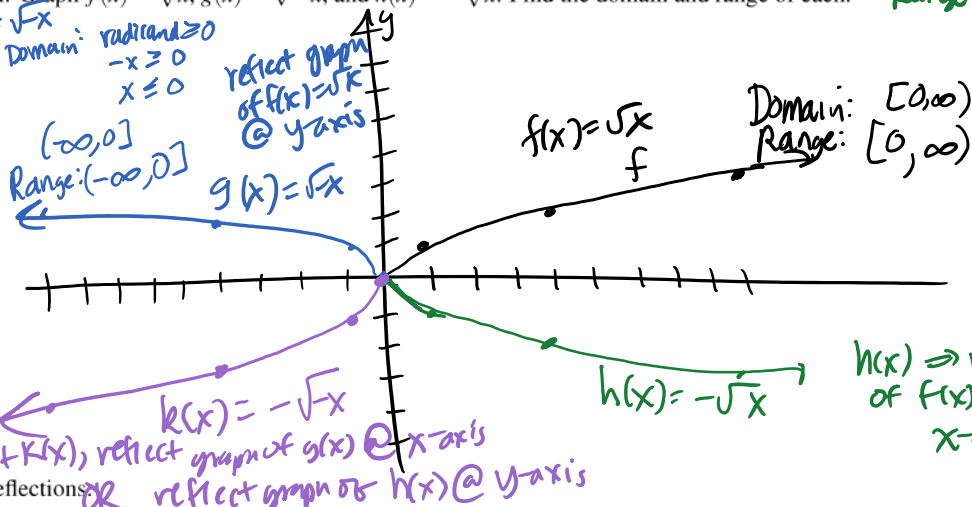
$y = \sqrt{x}$   $x \geq 0$

x	y
0	0
1	1
4	2
9	3
16	4
25	5

$g(x) = \sqrt{-x}$

x	y
0	0
-1	1
-4	2
-9	3
-16	4
-25	5

$g(x)$  is reflection of  $f(x)$  @ y-axis



Domain of  $h(x)$ :  $[0, \infty)$   
 Range:  $(-\infty, 0]$   
 $h(x) = -\sqrt{x}$

x	y
0	0
1	-1
4	-2
9	-3
16	-4

$h(x) \Rightarrow$  reflect graph of  $f(x) = \sqrt{x}$  @ x-axis

Reflections

- (a) To graph  $y = -f(x)$ : Reflect graph of  $y = f(x)$  about the x-axis
- (b) To graph  $y = f(-x)$ : Reflect graph of  $y = f(x)$  about the y-axis

Composition of fns: replace x w/ -x

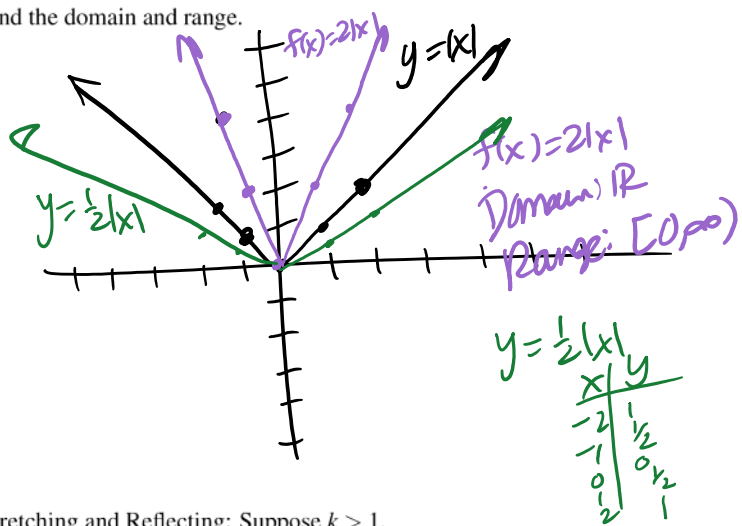
Ex: Graph  $f(x) = 2|x|$ . Find the domain and range.

$f(x) = 2|x|$  Domain:  $\mathbb{R}$

x	y
-2	4
-1	2
0	0
1	2
2	4

$y = |x|$

x	y
-2	2
-1	1
0	0
1	1
2	2



Vertical and Horizontal Stretching and Reflecting: Suppose  $k > 1$ .

- (a) To graph  $y = kf(x)$ : Stretch graph of  $y = f(x)$  vertically by a factor of  $k$
- (b) To graph  $y = \frac{1}{k}f(x)$ : Compress graph of  $y = f(x)$  vertically by a factor of  $k$
- (c) To graph  $y = f(kx)$ : Compress graph of  $y = f(x)$  horizontally by a factor of  $k$
- (d) To graph  $y = f(\frac{x}{k})$ : Stretch graph of  $y = f(x)$  horizontally by a factor of  $k$

Composition of fns

replace x w/ kx  
 x w/ x/k

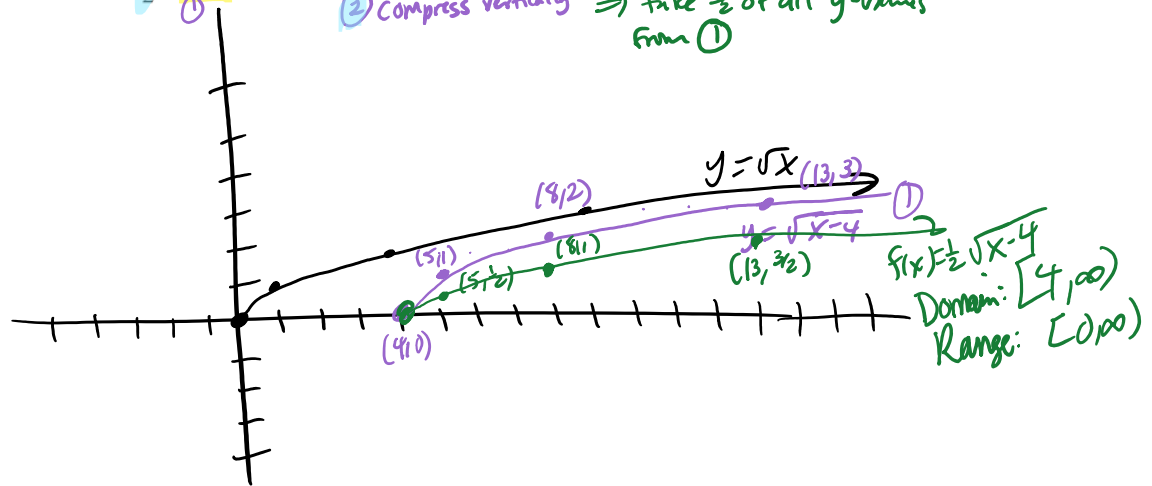
Parent fun:  $y = \sqrt{x}$

x	y
0	0
1	1
4	2
9	3
16	4

Ex: Graph  $f(x) = \frac{1}{2}\sqrt{x-4}$ . Find the domain and range.

① Shift graph of  $y = \sqrt{x}$  right 4

② Compress vertically  $\Rightarrow$  take  $\frac{1}{2}$  of all y-values from ①



③ Ex: Find a function that shifts the graph of  $f(x) = \sqrt{x}$  left 3 units, then shifts the graph down 2 units, then reflects it about the x-axis.

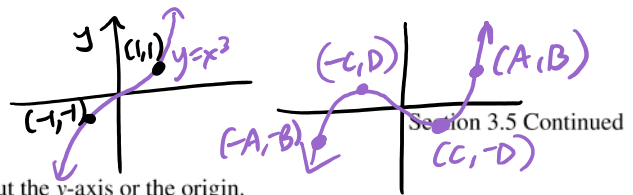
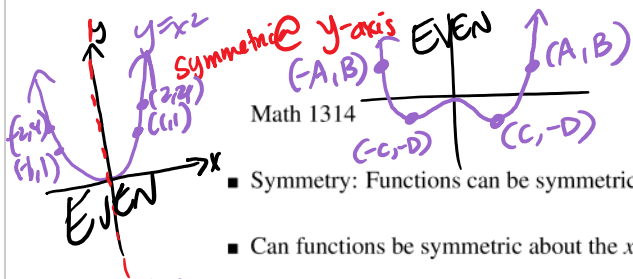
① Replace  $x$  with  $x+3$   
 need new  $y=0$  location at  $x=-3$

$$g(x) = -(\sqrt{x+3} - 2)$$

NOT THIS:

$$-\sqrt{x+3} - 2$$

- ① shift left 3
- ② reflect @ x-axis
- ③ shift down 2



Math 1314

- Symmetry: Functions can be symmetric about the y-axis or the origin.

- Can functions be symmetric about the x-axis?

No! Must pass Vertical Line Test

Strategy: replace x w/ -x.

- A function is even if  $f(-x) = f(x)$   
Even functions are symmetric about the y-axis
- A function is odd if  $f(-x) = -f(x)$   
Even functions are symmetric about the origin

Powers of x are even

Ex: Are the functions even, odd, or neither? Indicate symmetry.

(a)  $f(x) = -x^4 + 10x^2 - 9$

$$f(-x) = -(-x)^4 + 10(-x)^2 - 9$$

$$= -x^4 + 10x^2 - 9$$

$$= f(x) \checkmark$$

$$(-x)^4 = (-x)(-x)(-x)(-x)$$

$$= +x^4$$

$$(-x)(-x) = +x^2$$

$$(-x)(-x)(-x)(-x)(-x)(-x)(-x)(-x)$$

$$= -x^8$$

Powers of x are odd

(b)  $g(x) = 4x^7 - 5x^3 - 3x$

$$g(-x) = 4(-x)^7 - 5(-x)^3 - 3(-x)$$

$$= 4(-x^7) - 5(-x^3) + 3x$$

$$= -4x^7 + 5x^3 + 3x \neq g(x) \text{ Not even}$$

$$= -1(4x^7 - 5x^3 - 3x) = -g(x) \checkmark$$

Symmetry @ origin

EVEN  $\Rightarrow$  symmetry @ y-axis

(c)  $h(x) = -2x^3 + 4x^2 + 2$

$$h(-x) = -2(-x)^3 + 4(-x)^2 + 2$$

$$= -2(-x^3) + 4(+x^2) + 2$$

$$= 2x^3 + 4x^2 + 2 \neq h(x) \text{ Not even}$$

$$= -1(-2x^3 - 4x^2 - 2) \neq -h(x) \text{ not odd}$$

neither. Mix of even & odd powers

(d)  $j(x) = x^3 + x + 1$

$$j(-x) = (-x)^3 + (-x) + 1$$

$$= -x^3 - x + 1 \neq j(x) \text{ Not even}$$

$$= -1(x^3 + x - 1) \neq -j(x) \text{ not odd}$$

neither Mix of even & odd powers

(e)  $f(x) = 2x^4 - 2x^2 - 4$

$$f(-x) = 2(-x)^4 - 2(-x)^2 - 4$$

$$= +2x^4 - 2x^2 - 4 = f(x)$$

EVEN

- We will use the concept of even and odd in later sections to help us graph.

## COMBINING TRANSFORMATIONS

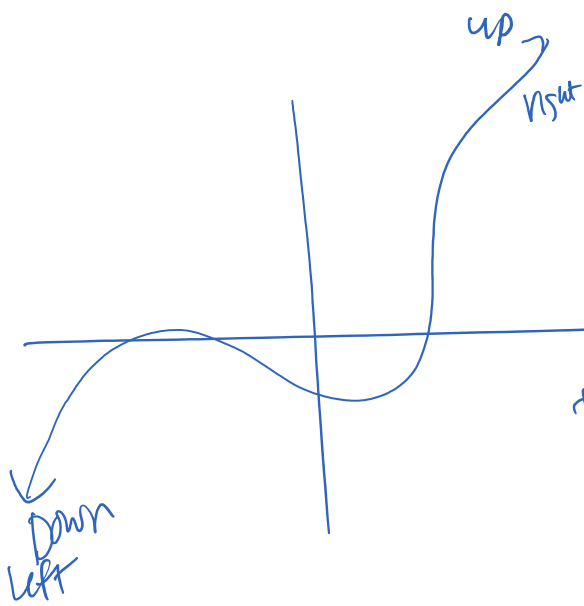
When combining vertical transformations written in the form  $af(x) + k$ , first vertically stretch by  $a$  and then vertically shift by  $k$ .

When combining horizontal transformations written in the form  $f(bx - h)$ , first horizontally shift by  $\frac{h}{b}$  and then horizontally stretch by  $\frac{1}{b}$ .

When combining horizontal transformations written in the form  $f(b(x - h))$ , first horizontally stretch by  $\frac{1}{b}$  and then horizontally shift by  $h$ .

Horizontal and vertical transformations are independent. It does not matter whether horizontal or vertical transformations are performed first.

$$\underline{af(x-h)}$$



Domain: [left, right]  
Range: [down, up]