

## Math 1314 – College Algebra

### Section 3.5 Transformation of Functions

Ex: Graph  $y = x^2$  and  $y = x^2 + 2$ . Give the domain and range of each. Also identify where each function is increasing or decreasing.

Ex: Graph  $y = (x - 2)^2$  and  $y = (x + 1)^2$ . Give the domain and range of each. Also identify where each function is increasing or decreasing.

- Vertical and Horizontal Shifts: Suppose  $k > 0$ .
  - (a) To graph  $y = f(x) + k$ : Shift graph of  $y = f(x)$   $k$  units upward
  - (b) To graph  $y = f(x) - k$ : Shift graph of  $y = f(x)$   $k$  units downward
  - (c) To graph  $y = f(x + k)$ : Shift graph of  $y = f(x)$   $k$  units left
  - (d) To graph  $y = f(x - k)$ : Shift graph of  $y = f(x)$   $k$  units right
- NOTE: When  $x$  has been replaced with another function, like  $x + \#$  or  $x - \#$  (think: composition of functions), we are looking at a horizontal shift.

Ex: Graph  $y = (x + 2)^2 - 3$ . Find the domain and range.

Ex: Graph  $y = |x - 1| - 2$ . Find the domain and range.

Ex: Graph  $f(x) = \sqrt{x}$ ,  $g(x) = \sqrt{-x}$ , and  $h(x) = -\sqrt{x}$ . Find the domain and range of each.

Reflections:

- (a) To graph  $y = -f(x)$ : Reflect graph of  $y = f(x)$  about the x-axis
- (b) To graph  $y = f(-x)$ : Reflect graph of  $y = f(x)$  about the y-axis

Ex: Graph  $f(x) = 2|x|$ . Find the domain and range.

■ Vertical and Horizontal Stretching and Reflecting: Suppose  $k > 1$ .

- (a) To graph  $y = kf(x)$ : Stretch graph of  $y = f(x)$  vertically by a factor of  $k$
- (b) To graph  $y = \frac{1}{k}f(x)$ : Compress graph of  $y = f(x)$  vertically by a factor of  $k$
- (c) To graph  $y = f(kx)$ : Compress graph of  $y = f(x)$  horizontally by a factor of  $k$
- (d) To graph  $y = f\left(\frac{x}{k}\right)$ : Stretch graph of  $y = f(x)$  horizontally by a factor of  $k$

Ex: Graph  $f(x) = \frac{1}{2}\sqrt{x-4}$ . Find the domain and range.

Ex: Find a function that shifts the graph of  $f(x) = \sqrt{x}$  left 3 units, then shifts the graph down 2 units, then reflects it about the  $x$ -axis.

- Symmetry: Functions can be symmetric about the  $y$ -axis or the origin.
- Can functions be symmetric about the  $x$ -axis?
- A function is even if  
Even functions are symmetric about the
- A function is odd if  
Odd functions are symmetric about the

Ex: Are the functions even, odd, or neither? Indicate symmetry.

(a)  $f(x) = -x^4 + 10x^2 - 9$

(b)  $g(x) = 4x^7 - 5x^3 - 3x$

(c)  $h(x) = -2x^3 + 4x^2 + 2$

(d)  $j(x) = x^3 + x + 1$

(e)  $f(x) = 2x^4 - 2x^2 - 4$

- We will use the concept of even and odd in later sections to help us graph.