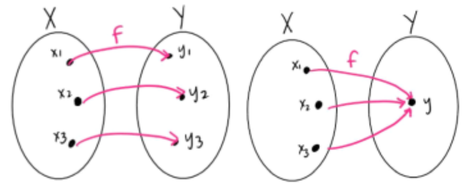
Math 1314 – College Algebra
Section 3.7 Inverse Functions

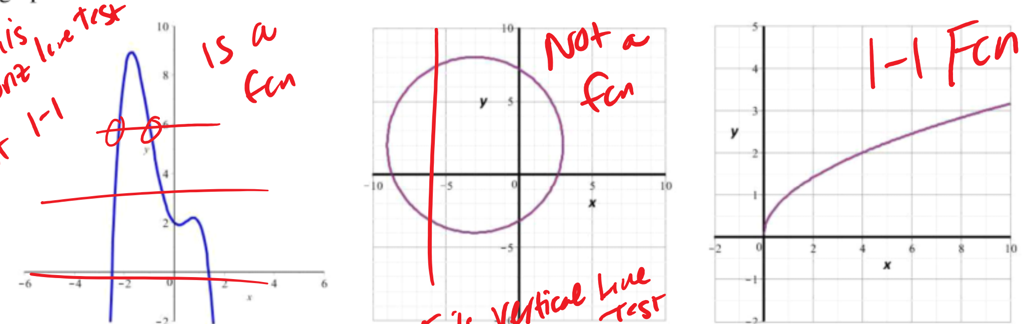
Recall:

> A function f from a set A to a set B is <u>one-to-one</u> if and only if different numbers in the domain of f have different outputs in the range of f.
> 1-1: If x_1 and x_2 are in the domain of f and $x_1 \neq x_2$, then $f(x_1) \neq f(x_2)$.



- We can test to determine if a graph represents a function using the Horizontal Line Test. Are the following graphs of 1-1 functions?

Fails Horiz line test
Not 1-1
is a fcn



Not a fcn
Fails Vertical Line Test

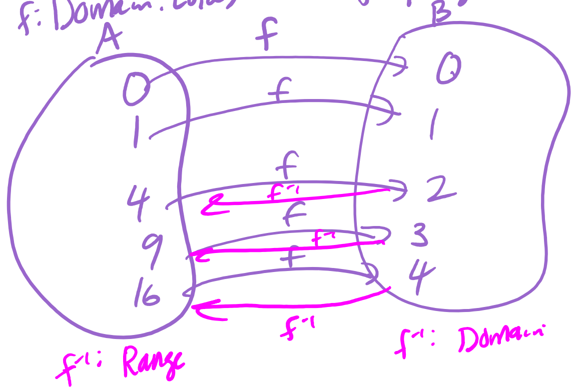
1-1 Fcn

- Consider $f(x) = \sqrt{x}$.

f: Domain: radicand ≥ 0
$x \geq 0$
$[0, \infty)$

f: Domain: $[0,\infty)$ f: Range: $[0,\infty)$ f is a mapping

f^{-1}



$2^2 = 4$
$3^2 = 9$
$4^2 = 16$

- If f is a 1-1 function with domain A and range B, then the inverse of f, $\boxed{f^{-1}}$, has domain <u>B</u> and range <u>A</u>.
- We write: $f^{-1}(x)$ f inverse of x
- NOTE:

$$f^{-1}(x) \neq \frac{1}{f(x)}$$

This is a 1-1 fcn

- Finding the Inverse of a 1-1 Function:

1. Let $y = f(x)$
2. Interchange x & y.
3. Solve for y.
4. $y = f^{-1}(x)$

Ex: Find the inverse of $f(x) = \frac{2}{3}x + 4$.

$y = \frac{2}{3}x + 4$

Switch x&y:

$x = \frac{2}{3}y + 4$

$\left(\frac{3}{2}\right)(x-4) = \left(\frac{3}{2}\right)\frac{2}{3}y$

$\frac{3}{2}(x-4) = y$

$y = \frac{3}{2}(x-4)$ or $y = \frac{3}{2}x - 6$

$\boxed{f^{-1}(x) = \frac{3}{2}x - 6}$

$\frac{3}{\cancel{2}}\left(-\frac{\cancel{4}^{2}}{1}\right)$

$f(x) = \frac{2}{3}x + 4$

y-int: (0,4) $m = \frac{2}{3}$

x-int: Set y=0:

$0 = \frac{2}{3}x + 4$

$\left(\frac{3}{2}\right)(-4) = \left(\frac{3}{2}\right)\frac{2}{3}x$

$-6 = x$

(-6,0)

$f^{-1}(x) = \frac{3}{2}x - 6$ $m = \frac{3}{2}$

y-int: (0,-6)

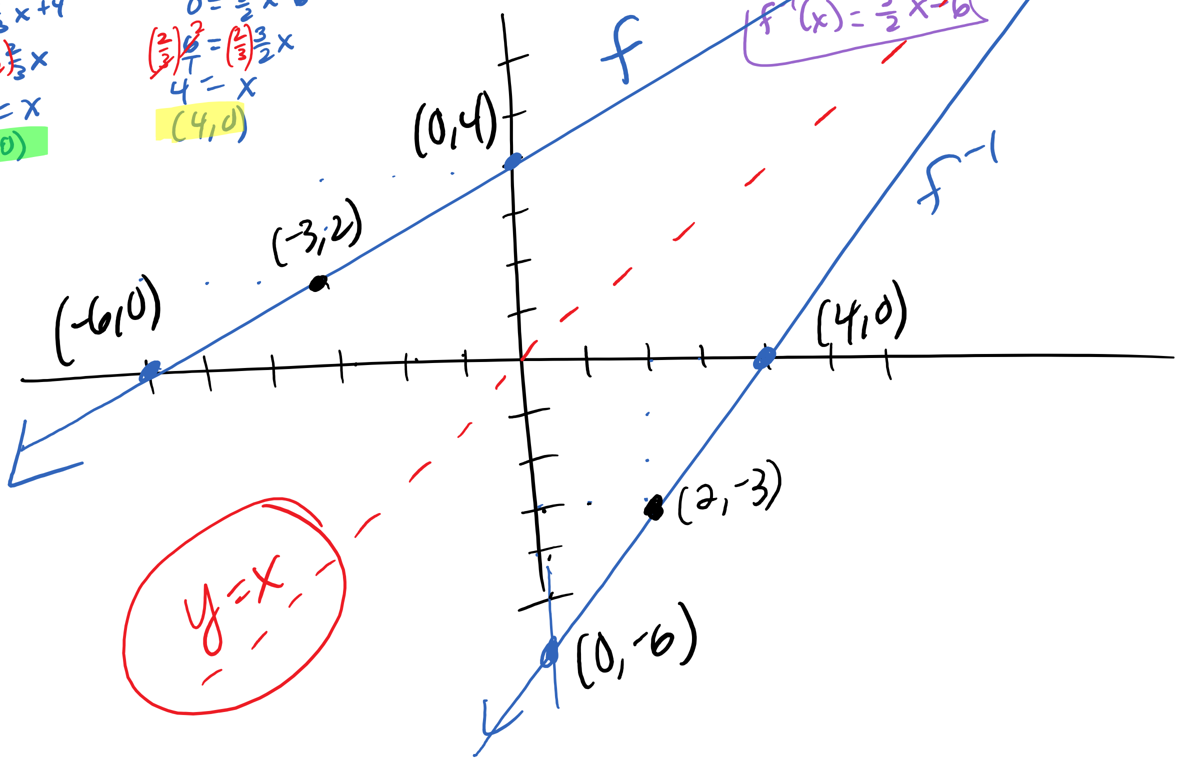
x-int: Set y=0

$0 = \frac{3}{2}x - 6$

$\left(\frac{2}{3}\right)\frac{6}{1} = \left(\frac{2}{3}\right)\frac{3}{2}x$

$4 = x$

(4,0)



$f(x) = \frac{2}{3}x + 4$ $\quad f^{-1}(x) = \frac{3}{2}\boxed{x} - 6$

$(f \circ f^{-1})(x) = f(f^{-1}(x)) = f\left(\frac{3}{2}x - 6\right) = \frac{2}{3}\left(\frac{3}{2}x - 6\right) + 4 = x - \frac{2}{3}\left(\frac{6}{1}\right) + 4$

$= x - 4 + 4 = x$

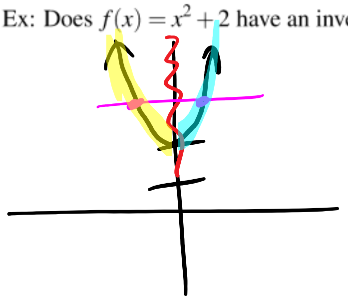
$(f^{-1} \circ f)(x) = f^{-1}(f(x)) = f^{-1}\left(\frac{2}{3}x + 4\right) = \frac{3}{2}\left(\frac{2}{3}x + 4\right) - 6 = x + 6 - 6 = x$

$\frac{3}{2}\left(\frac{4}{1}\right)$

$(f \circ f^{-1})(37) = 37$ $\qquad$ $(f^{-1} \circ f)(119.3) = 119.3$

- NOTE: $(f \circ f^{-1})(x) = x$ and $(f^{-1} \circ f)(x) = x$ if f & f^{-1} are inverses
- Summary of Cool Stuff about Inverses: f & f^{-1}:
  - Domain of f = Range of f^{-1} AND Range of f = Domain of f^{-1}
  - If (a,b) is on the graph of f, then (b,a) is on graph of f^{-1}
  - Graphs of f and $f^{-1}(x)$ are symmetric @ $y=x$.
  - $(f \circ f^{-1})(x) = x$ AND $(f^{-1} \circ f)(x) = x$

Ex: Does $f(x) = x^2 + 2$ have an inverse? NO. Fails Horiz Line Test.



Where is f one-to-one and non-increasing? Const or decr.

$(-\infty, 0]$

One-to-one and non-decreasing? Incr. or Constant

$[0, \infty)$

Ex: Does $f(x) = x^2 + 2, \quad x \le 0$ have an inverse? YES! If so, find $f^{-1}(x)$, the domain and range of each, and graph both.

$f(x) = x^2 + 2, \ x \le 0$

$y = x^2 + 2, \ x \le 0$

Switch x & y:

$x = y^2 + 2, \ y \le 0$

Solve for y:

$x - 2 = y^2, \quad y \le 0$

$\pm\sqrt{x-2} = \sqrt{y^2}, \ y \le 0$

$\pm\sqrt{x-2} = y, \ y \le 0$

use $-\sqrt{}$ b/c y is 0 or neg

$y = -\sqrt{x-2}$ $\quad$ $y \le 0$

$f^{-1}(x) = -\sqrt{x-2}$

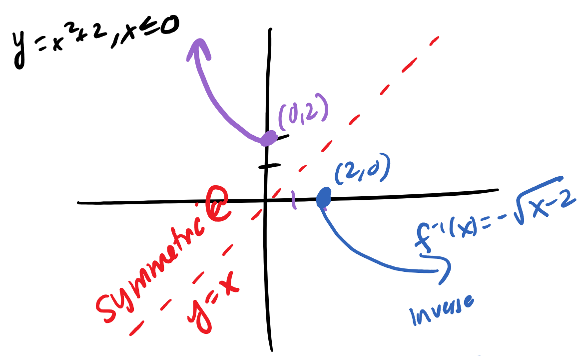
Domain: radicand ≥ 0

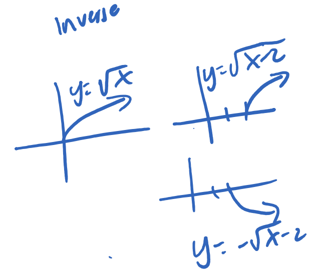
$x - 2 \ge 0$

$x \ge 2$

f :Domain: $(-\infty, 0]$ $\qquad$ f^{-1} :Domain: $[2, \infty)$

f :Range: $[2, \infty)$ $\qquad$ f^{-1} :Range: $(-\infty, 0]$

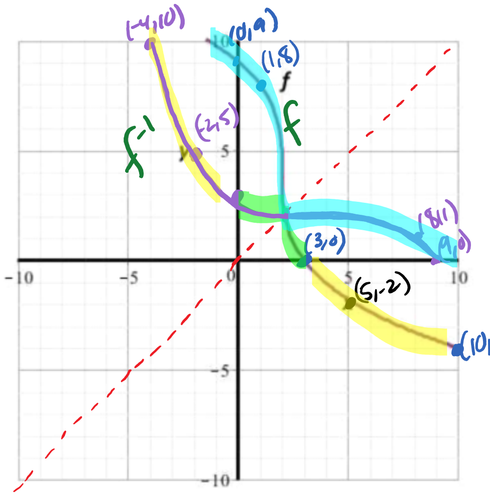




$y = f(x) \iff f^{-1}(y) = x$

$8 = f(1) \iff f^{-1}(8) = 1$

Ex: Given the graph of the function below, answer the following questions.



(a) Is f one-to-one? yes! Passes VLT & HLT (fcn ✓, 1-1 ✓)

(b) Find $f(10) = -4$

(c) Find $f^{-1}(8) = 1$ $f(?) = 8$, $f(1) = 8$

(d) Find $f(0) = 9$

(e) Find $f^{-1}(0) = 3$ $f(3) = 0$

(f) Find $f^{-1}(-4) = 10$ $f(10) = -4$

(g) Solve $f^{-1}(x) = 5$ $x = -2$ $f(5) = -2$

(h) Find $(f \circ f^{-1})(0) = f(f^{-1}(0)) = f(3) = 0$

(i) Find $(f^{-1} \circ f)(0) = 0$

$f^{-1}(f(0)) = f^{-1}(9) = 0$

(j) Plot $f^{-1}(x)$ on the same set of axes.

Ex: If $f(x) = \dfrac{2x}{x-3}$, find f^{-1}. Find the domain and range of both.

f: Domain: denom ≠ 0

$x - 3 \neq 0$

$x \neq 3$

f :Domain: $(-\infty,3) \cup (3,\infty)$ f^{-1} :Domain: $(-\infty,2) \cup (2,\infty)$

f :Range: $(-\infty,2) \cup (2,\infty)$ f^{-1} :Range: $(-\infty,3) \cup (3,\infty)$

~~[illegible]~~ 3

Find f^{-1}: $y = \dfrac{2x}{x-3}$

Switch x & y: $x = \dfrac{2y}{y-3}$ Solve for y

$x(y-3) = \dfrac{2y}{\cancel{(y-3)}}\cancel{(y-3)}$

$x(y-3) = 2y$

$xy - 3x = 2y$

$-2y \quad -2y$

$xy - 2y - 3x = 0$

$+3x \quad +3x$

$xy - 2y = 3x$

$\dfrac{y(x-2)}{(x-2)} = \dfrac{3x}{(x-2)}$

$y = \dfrac{3x}{x-2}$

$f^{-1}(x) = \dfrac{3x}{x-2}$

OR

$xy - 3x = 2y$

$-xy \quad -xy$

$-3x = 2y - xy$

$-3x = y(2-x)$

$\dfrac{-3x}{(2-x)} = \dfrac{y(2-x)}{(2-x)}$

$\dfrac{-3x}{2-x} = y$

$\dfrac{(-1)(3x)}{(2-x)} = \dfrac{3x}{(-1)(2-x)} = \dfrac{3x}{-2+x}$

Different ex:

$3y - 5y = 14$

$(3-5)y = 14$

$\dfrac{-2y}{-2} = \dfrac{14}{-2}$

$y = \dfrac{14}{-2}$

Domain of f^{-1}: denom ≠ 0

$x - 2 \neq 0$

$x \neq 2$

~~[illegible]~~ 2

$(-\infty,2) \cup (2,\infty)$