

Math 1314 – College Algebra

Section 4.1-4.2 Linear Functions/Modeling with Linear Functions

RECALL:

- The slope, m , of the line containing the points $P(x_0, y_0)$ and $Q(x_1, y_1)$ is: $m = \frac{y_1 - y_0}{x_1 - x_0} = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x}$
- The point-slope form of the line with slope m containing the point (x_0, y_0) is the equation: $(y - y_0) = m(x - x_0)$
- The slope-intercept form of the line with slope m and y -intercept $(0, b)$ is the equation: $y = mx + b$
- A linear function is a function of the form $f(x) = mx + b$, where m and b are real numbers, with $m \neq 0$. The domain of a linear function is $(-\infty, \infty)$.
- A constant function is a function of the form $f(x) = b$, where b is a real number. The domain of a constant function is $(-\infty, \infty)$.
- Two lines are parallel if and only if their slopes are identical.
- Two lines are perpendicular if and only if the product of their slopes is -1 . (or their slopes are negative reciprocals of each other).

$$f(x) = y \qquad y - y_1 = m(x - x_1)$$

Ex: Find a linear equation for $f(x)$ if $f(100) = 3.92$ and $f(0) = 1$.

$$m = \frac{1 - 3.92}{0 - 100} = \frac{-2.92}{-100} = +.0292$$

$(100, 3.92)$ $(0, 1) \leftarrow y\text{-int}$ $y = mx + b$
 $b = 1$ $y = 0.0292x + 1$
 $f(x) = .0292x + 1$

$x=0$ depth=0
 y is ft
 depth=

Ex: The pressure p of water on a diver's body is a linear function of the diver's depth, x . At the water's surface, the pressure is 1 atmosphere. At a depth of 100 ft, the pressure is approximately 3.92 atmospheres.

(a) Find the linear function that relates p to x .

x, p y depends on x (x, y)
 p depends on x (x, p)

$(100, 3.92)$ $(0, 1)$
 ∴ see work above
 $f = .0292x + 1$

$p(x) = .0292x + 1 \leftarrow$ at a depth of x ft, pressure is $p(x)$

(b) Compute the pressure at a depth of 10 fathoms. (Note: 1 fathom = 6 ft.)

10 fathoms = 60 ft

$$p(60) = .0292(60) + 1$$

$$= 2.752 \text{ atm (atmospheres)}$$



Ex: A certain gas reserve held about 19 billion cubic feet of the gas in 2010 and is being depleted by about 1.1 billion cubic feet each year.

- (a) Give a linear equation for the remaining gas reserves, R , in terms of t , the number of years since 2010. (Let R be measured in billions of cubic feet.)

in terms of R (x, y)

$(2010, 19)$ $(0, 19)$ $(1, 19 - 1.1)$

$$m = \frac{\Delta R}{\Delta t} = \frac{-1.1 \text{ billion ft}^3}{1 \text{ yr}} = -1.1$$

$$y = mx + b$$

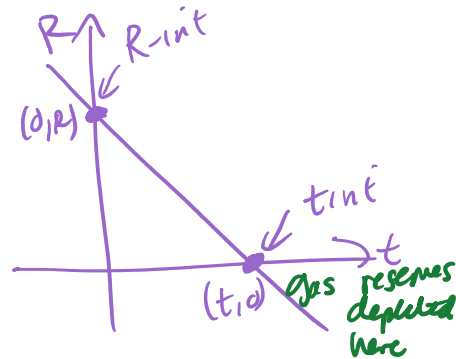
$$R(t) = -1.1t + 19 = 19 - 1.1t$$

- (b) What does the t -intercept represent?
here $R=0$.

time when gas reserves have been depleted

- (c) What does the R -intercept represent?
here, $t=0 \Rightarrow 2010$ is the year

Amt of gas in reserve in 2010



- (d) In 2015, what will the gas reserves be? (Round your answer to one decimal place.)

$2015 - 2010 = 5$ years since 2010

$R(5) = -1.1(5) + 19 = 13.5$

In 2015, gas reserves will be 13.5 billion cubic feet.

- (e) If the rate of depletion doesn't change, in what year will the gas reserve be depleted? (Round your answer down to the nearest year.)

Find t_{int} : $R=0$

$0 = -1.1t + 19$

$-19 = -1.1t$

$1.1t = 19$

$t = \frac{19}{1.1} \approx 17.27 \Rightarrow$ use 17 years.

2010 + 17 years \Rightarrow 2027 is estimated year reserves will be depleted.

- (f) Find a reasonable domain for the function.

$[0, 17]$

Start in 2010. reserves are depleted in 2027

- (g) Find a reasonable range for the function.

$[0, 19]$

