Math 1314 – College Algebra
Section 5.1 Quadratic Functions

$(-\frac{b}{2a}, y)$ vertex

(turning point) vertex

- A quadratic function is a second-degree polynomial function in one variable. Suppose $a, b, c \in \mathbb{R}, a \neq 0$.

Form	Vertex	Axis of Symmetry	If $a > 0$	If $a < 0$
General: $f(x) = ax^2 + bx + c$	$\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$	$x = \frac{-b}{2a}$	$++$ ∪	$--$ ∩
Standard: $f(x) = a(x-h)^2 + k$	(h, k)	$x = h$		

- Domain: $\mathbb{R}$

$a\,(x-h)^2 + k$

$f(x) = -2(x-(-1))^2 + 3$

② ① ③

Ex: Find the vertex and graph $f(x) = -2(x+1)^2 + 3$. Give the domain and range. Give the x- and y-intercepts, if there are any.

Parent fcn: $y = x^2$

① Shift left 1
② double y-values, flip @ ~~x-axis~~
③ Shift up 3

Opens down ∩ $a = -2$

vertex: $f(x) = a(x-h)^2 + k$

$f(x) = -2(x+1)^2 + 3$

$a = -2$
$h = -1$
$k = 3$

$(x-h) = (x+1)$

$\cancel{x} - h = \cancel{x} + 1$

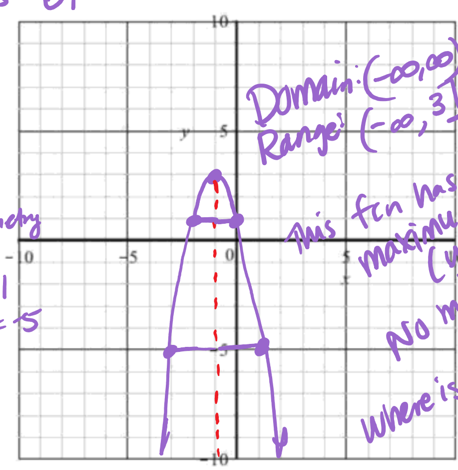
$-h = 1$

$h = -1$

vertex: $(h, k) = (-1, 3)$

$f(x) = -2(x+1)^2 + 3$

x	y
-3	-5
-2	1 by symmetry
vertex -1	3
0	$-2(1)^2+3 = 1$
1	$-2(2)^2+3 = -5$



Domain: $(-\infty, \infty)$ or $\mathbb{R}$

Range: $(-\infty, 3]$

This fcn has a maximum of 3. (y-value)

No minimum

Where is the max? at $(-1, 3)$

$y = ax^2 + bx + c$ general form

$y = a(x-h)^2 + k$ Std form

Ex: Find the standard form and general form of the parabola with vertex $(2, 9)$ that passes through the point $(5, 27)$.

Std form: $y = a(x-2)^2 + 9$

Need to find a. Use the other point! $(x = 5 \quad y = 27)$

$27 = a(5-2)^2 + 9$

$27 = a(3)^2 + 9$

$27 = 9a + 9$

$18 = 9a$

$a = 2$

now plug a into std form

Solve for a

$\boxed{f(x) = 2(x-2)^2 + 9}$ std form

Vertex is min. axis of sym $x = 2$

$f(x) = 2(x-2)(x-2) + 9$

$= 2(x^2 - 4x + 4) + 9$

$= 2x^2 - 8x + 8 + 9$

$\boxed{f(x) = 2x^2 - 8x + 17}$

general form vertex: $x = \frac{-b}{2a} = \frac{+8}{2(2)}$

Ex: Find the general form of the parabola with intercepts (2, 0), (−1, 0), and (0, −6).

x-int y-int

$f(x) = \cancel{a(x-h)^2} + K$ or $f(x) = ax^2 + bx + c$ when $x = 0$, $y = -6$

$y = a(x-2)(x-(-1))$
$y = a(x-2)(x+1)$
$-6 = a(0-2)(0+1)$
$-6 = a(-2)(1)$
$-6 = -2a$
$3 = a$

$f(x) = 3(x-2)(x+1)$
$f(x) = 3(x^2+x-2x-2)$
$f(x) = 3(x^2-x-2)$
$\boxed{f(x) = 3x^2-3x-6}$

vertex: $x = -\frac{b}{2a} = \frac{\cancel{3}}{2(\cancel{3})} = \frac{1}{2}$

y-coord: $f(\tfrac{1}{2}) = 3(\tfrac{1}{2}-2)(\tfrac{1}{2}+1)$
$= 3(\tfrac{1}{2}-\tfrac{4}{2})(\tfrac{1}{2}+\tfrac{2}{2})$
$= \frac{3}{1}(-\tfrac{3}{2})(\tfrac{3}{2})$
$= -\frac{27}{4}$

$f(\tfrac{1}{2}) = 3(\tfrac{1}{2})^2 - 3(\tfrac{1}{2}) - 6$ vertex: $(\tfrac{1}{2}, -\tfrac{27}{4})$

- There are times when we are given one form of a quadratic and it is useful to convert to the other form.

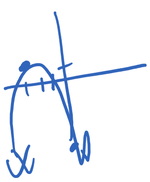
Ex: Convert $f(x) = -x^2 - 6x - 8$ to standard form. $f(x) = a(x-h)^2 + K$

Complete the square on x!!!

$f(x) = -x^2 - 6x \quad -8$
$= -1(x^2+6x+0) - 8$
$= -1(x^2+6x+(3)^2-(3)^2) - 8$
$= -1((x+3)^2 - 9) - 8$
$= -1(x+3)^2 + 9 - 8$
$\boxed{f(x) = -1(x+3)^2 + 1}$ std form

vertex: $(-3, 1)$
$a = -1$

- factor x terms so coeff of x^2 is 1: factor out a −1
- Complete sq on terms in parentheses
  - $\frac{1}{2}$ coeff of x: $\frac{1}{2}(6) = 3$
  - Square it: 3^2
  - add & subtract 3^2 inside parens.
- factor 1st 3 terms in parens.
- Distribute −1 into parens.



OR

$f(x) = -x^2 - 6x - 8$ $a = -1$ $b = -6$ $c = -8$

vertex: $x = -\frac{b}{2a} = \frac{6}{2(-1)} = -3$ vertex $(-3, 1)$

y-coord: $f(-3) = -(-3)^2 - 6(-3) - 8$
$= -9 + 18 - 8 = 1$ $h = -3$ $k = 1$

$f(x) = a(x-h)^2 + K$
$f(x) = -1(x-(-3))^2 + 1 = -1(x+3)^2 + 1$

$g(x) = 2x(x+8) + 33$ $\quad g(-3) = -6(5) + 33 = 3$ $\quad g(-3) = 2(-3)^2 + 16(-3) + 33$

Ex: Find the vertex and sketch $g(x) = 2x^2 + 16x + 33$. Give the domain and range. Give the x- and y-intercepts, if there are any.

complete sq:

$g(x) = 2(x^2 + 8x + 0) + 33$

$= 2(x^2 + 8x + 4^2 - 4^2) + 33$

$= 2((x+4)^2 - 16) + 33$

$= 2(x+4)^2 - 2(16) + 33$

$= 2(x+4)^2 - 32 + 33$

$g(x) = 2(x+4)^2 + 1$

$\frac{1}{2}(8) = 4$

or: $g(x) = 2x^2 + 16x + 33$

vertex: $x = \frac{-b}{2a} = \frac{-16}{2(2)} = -4$

y-coord: $f(-4) = 2(-4)^2 + 16(-4) + 33$

$= 2(16) + 16(-4) + 33$

$= \cancel{2(16)[-1 + 2] + 33}$

$= \cancel{32(-1) + 33} = 1$

$a = 2 \quad h = -4 \quad k = 1$

$a(x-h)^2 + k$

$g(x) = 2(x+4)^2 + 1$

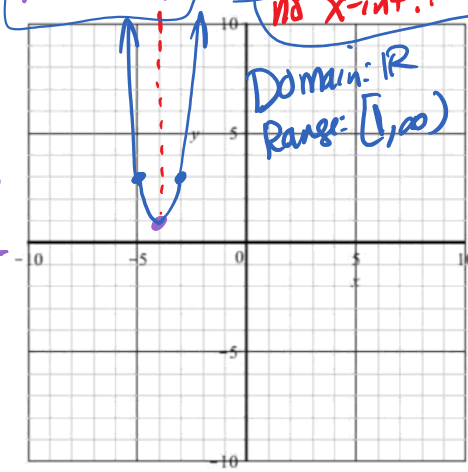
Vertex $(-4, 1)$ opens up

No x-int!! y-int: $(0, 33)$

Domain: $\mathbb{R}$

Range: $[1, \infty)$

x	y
-5	3
-4	1
-3	3

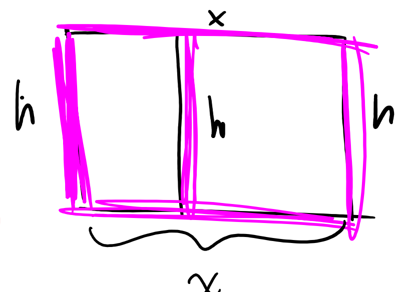


Ex: A rancher wants to enclose a rectangular partitioned corral with 1800 feet of fencing. What dimensions of the corral would enclose the largest possible area? Find the maximum area.

Max: $A = (\text{length})(\text{width}) = xh$

Need a fn in terms of only one variable.

Need to get rid of a variable $\Rightarrow$ write $h = x$ stuff & sub back in



Fencing: 1800 ft total

$1800 = x + h + x + h + h$

$1800 = 2x + 3h$ Solve for h

$-2x \quad -2x$

$1800 - 2x = 3h$

$\frac{1800 - 2x}{3} = \frac{3h}{3}$

$\frac{1800}{3} - \frac{2}{3}x = \frac{1800 - 2x}{3} = h$

Area: $x\left(600 - \frac{2}{3}x\right)$

$A(x) = 600x - \frac{2}{3}x^2$

$A(x) = -\frac{2}{3}x^2 + 600x$

$(x, A(x))$ graph is a parabola opening down

Notice the y-value is the largest at the vertex

Here, our y-value represents area of corral.

Find the vertex!

Vertex: $x = \frac{-b}{2a}$

$x = \frac{-600}{2\left(-\frac{2}{3}\right)} = \frac{(-600)}{\left(-\frac{4}{3}\right)}$

$= \left(\frac{+600}{1}\right) \cdot \left(+\frac{3}{4}\right) = 450\text{ ft}$

$h = 600 - \frac{2}{3}(450)$

$= 600 - \frac{900}{3}$

$= 600 - 300 = 300$

Dimensions:

Max area: $(300\text{ft})(450\text{ft}) = 135{,}000\text{ ft}^2$

$$\left(\frac{\$1.30}{\text{person}}\right)(\#\text{ of } \cancel{\text{persons}}) = \$$$

every penny increase results in 1000 less riders.

Ex: The Municipal Transit Authority serves 150,000 commuters daily when the fare is $1.30. Market research has determined that every penny decrease will result in 1,000 new riders. What is the maximum income?

(price per person)(# of people) = $

orginal: $(1.30)(150{,}000)$

decr by penny: $(1.30-.01)(150{,}000+1000)$

decr by 2 pennies: $(1.30-.01-.01)(150{,}000+1000+1000) = (1.30-2(.01))(150{,}000+2(1000))$

decr by 3 pennies: $(1.30-.01-.01-.01)(150{,}000+1000+1000+1000)$
$= (1.30-3(.01))(150{,}000+3(1000))$

decr by 4 pennies: $(1.30-4(.01))(150{,}000+4(1000))$

⋮

decr by x pennies: $(1.30-x(.01))(150{,}000+x(1000))$

income: $I(x) = (1.30-.01x)(150{,}000+1000x)$

use x = -10. $(1.30-.01(-10))(150{,}000+1000(-10))$

$150{,}000-10(1000) = 140{,}000$ riders

Income: $(\$1.40)(140{,}000) = \$196{,}000$

$I(x) = 195{,}000 + 1300x - 1500x - 10x^2 = -10x^2 - 200x + 195{,}000$

x = # of penny decrease

If x = -#, INCREASE fare

vertex $(x, I(x))$ (# of penny decr, income)

Max income at vertex. $x = -\frac{b}{2a} = \frac{200}{2(-10)} = -10$

what is fare that gives max income: ~~$1.20~~ $1.40

Ex: Find two numbers whose sum is 23 and whose product is a maximum.

x = one #
y = other #

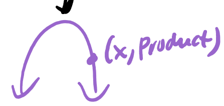
x + y = 23

Max product: x(y)

Too many variables. get rid of y.
Need y = x stuff & sub back in
Use x + y = 23 ⇒ y = 23 − x

~~1. 22~~
~~2. 21~~
~~3. 20~~

max product occurs at the vertex



Max product: $P(x) = x(23-x) = 23x - x^2$
$P(x) = -x^2 + 23x$

vertex: $x = \frac{-b}{2a}$

$x = \frac{-23}{2(-1)} = \frac{23}{2} = 11.5$

$y = 23 - x$

$y = \frac{23}{1} - \frac{23}{2} = 23\left(1-\frac{1}{2}\right) = \frac{23}{1}\left(\frac{1}{2}\right) = \frac{23}{2}$

$\boxed{\text{The two numbers are } 11.5 \,\&\, 11.5 \text{ or } \frac{23}{2} \,\&\, \frac{23}{2}.}$