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# Math 1314 – College Algebra
## Sections 5.2-5.3 Power Functions and Polynomial Functions/Graphs of Polynomial Functions

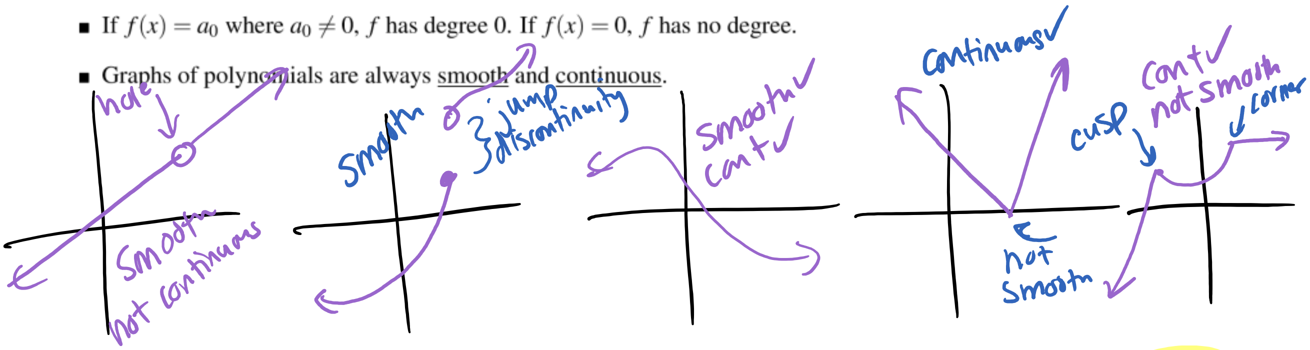
- A polynomial function in one variable (let's use $x$) is $f(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \ldots + a_1 x + a_0$.
  - Degree is $n$, highest power of $x$ ▪ Leading term is $a_n x^n$ ▪ Leading coefficient is $a_n$ ▪ Constant term is $a_0$
- Looks like: Sum and/or difference of terms whose variable $x$ has whole number powers. Coefficients are real numbers.

  $f(x) = 5x^0$

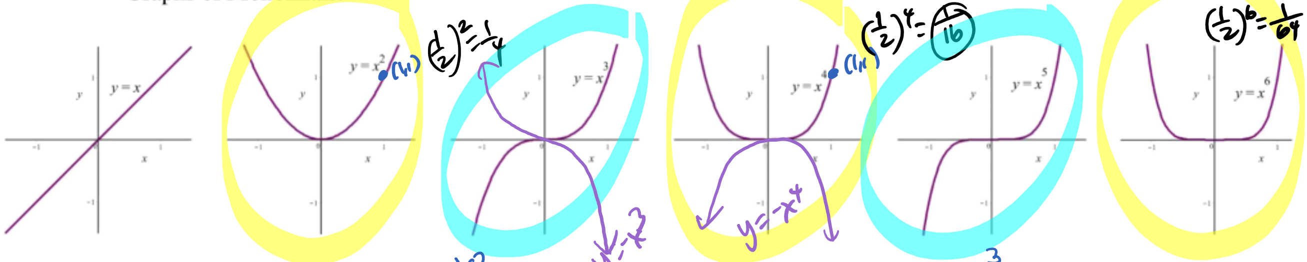
- Examples of polynomials:

| Function | Example | Degree | Graph |
|---|---|---|---|
| Constant Fcn | $f(x) = 5$ | 0 | |
| Linear Fcn | $g(x) = 3x + 11$ | 1 | |
| Quadratic Fcn | $h(x) = 2x^2 - 17x + 5$ | 2 | |

- If $f(x) = a_0$ where $a_0 \neq 0$, $f$ has degree 0. If $f(x) = 0$, $f$ has no degree.
- Graphs of polynomials are always smooth and continuous.




Graphs of Monomials:




- $x^n$ has the same general shape as __$x^2$__ when $n$ is even and the same general shape as __$x^3$__ when $n$ is odd.
- End behavior: What happens to $y$-values as $x$ becomes large + and large −
  - $x \to \infty$: large in the positive direction
  - $x \to -\infty$: large in the negative direction
- End behavior is determined by the term with the highest power of $x$.
  - For $x^{\text{even}}$, tails point in same direction
  - For $x^{\text{odd}}$, tails point in opposite directions
- Given $P(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \ldots + a_1 x + a_0$,

| $P$ has odd degree: | | $P$ has even degree: | |
|---|---|---|---|
| Leading Coefficient +: | Leading Coefficient −: | Leading Coefficient +: | Leading Coefficient −: |
| ? | ? | ? | ? |

Ex: Find the end behavior:

(a) $f(x) = 3x^4 + 4x^3 + 5x^2 + 6x + 7$

left: as $x \to -\infty$, $y \to \infty$ up

right: as $x \to \infty$, $y \to \infty$ up

$y = x^3$

(b) $g(x) = -19x^{15} - 2x^4 + 3x^3 + 6x - 2$

odd ⇒ tails are opposite

left: as $x \to -\infty$, $y \to \infty$ up

right: as $x \to \infty$, $y \to -\infty$ down

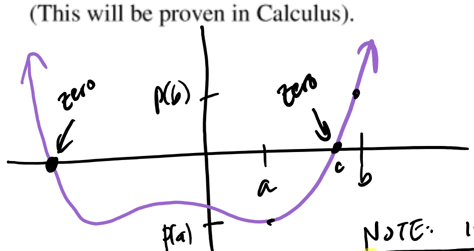
(c) $h(x) = -8x^6 + 3x^4 - 2x^3 - 17$

as $x \to -\infty$, $y \to -\infty$

as $x \to \infty$, $y \to -\infty$

Intermediate Value Theorem for Polynomials: If $P$ is a polynomial function and $P(a)$ and $P(b)$ have opposite signs, then there exists at least one value $c$ between $a$ and $b$ such that $P(c) = 0$.
(This will be proven in Calculus).

$(x, f(x))$ input output; y-values; a & b are x-values; x-values




IMPORTANT: Between any two successive zeros, the values of a polynomial are all + or all −

y-values

(Between any 2 successive zeros, the graph of a polynomial lies entirely above or below the x-axis

NOTE: If $(a,0)$ is x-int, $x=a$ is a zero

Ex: Graph $y = -x^4 + 10x^2 - 9 = -1(x-3)(x+3)(x-1)(x+1)$. Consider symmetry.

End behavior: as $x \to -\infty$, $y \to -\infty$; as $x \to +\infty$, $y \to -\infty$

y-int: $(0, -9)$

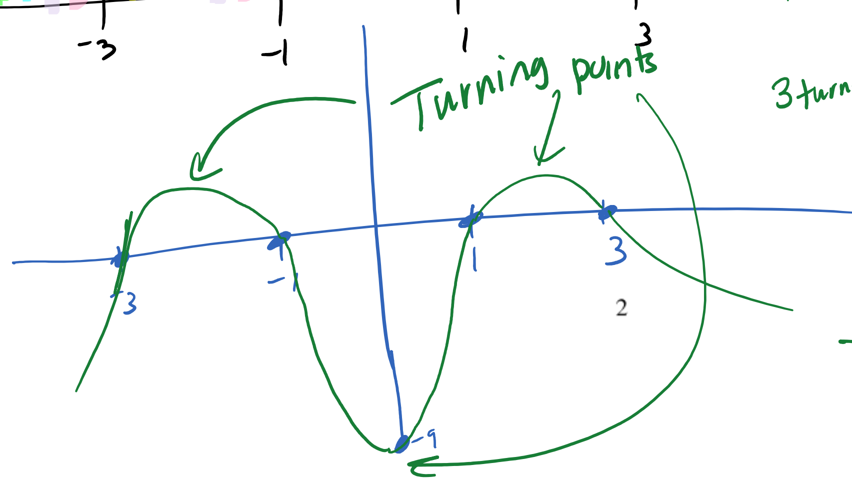
$f(-x) = -(-x)^4 + 10(-x)^2 - 9$
$= -x^4 + 10x^2 - 9 = f(x)$
EVEN
symmetric @ y-axis

$y = -1(x-3)(x+3)(x-1)(x+1)$

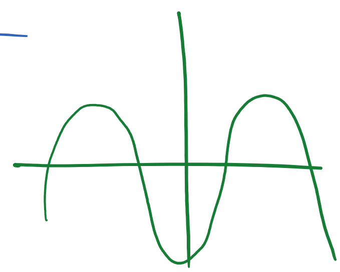
Zeros: $0 = -1(x-3)(x+3)(x-1)(x+1)$
$x = 3,\ x = -3,\ x = 1,\ x = -1$

| − | + | − | + | − |
|---|---|---|---|---|
| (−)(−)(−)(−)(−) | (−)(−)(+)(−)(−) | (−)(−)(+)(−)(+) | (−)(−)(+)(+)(+) | (−)(+)(+)(+)(+) |

−3, −1, 1, 3




3 turning points



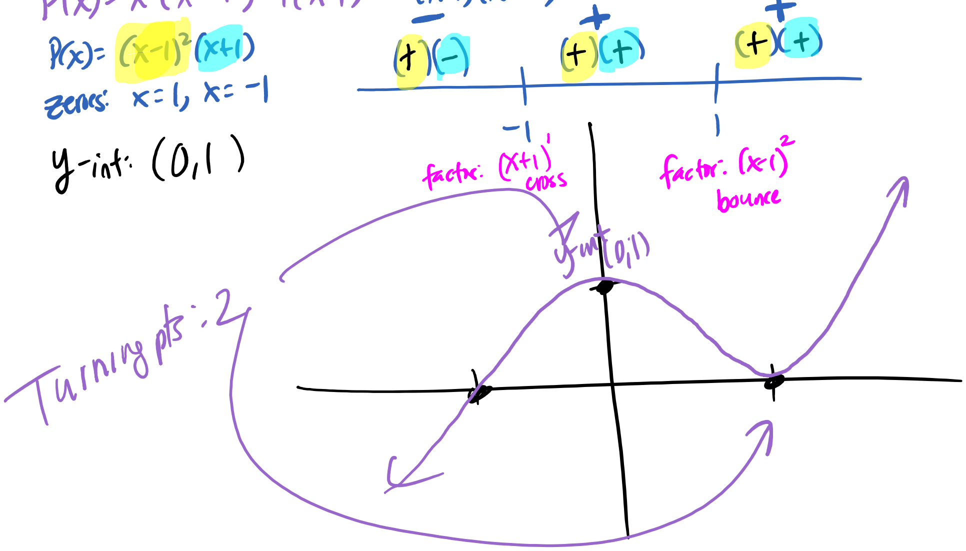
Ex: Let $P(x) = x^3 - x^2 - x + 1$. Find zeros and sketch.

$P(x) = x^2(x-1) - 1(x-1) = (x-1)(x^2-1) = (x-1)(x-1)(x+1) = (x-1)^2(x+1)$

$P(x) = (x-1)^2(x+1)$

zeros: $x = 1$, $x = -1$

y-int: $(0,1)$



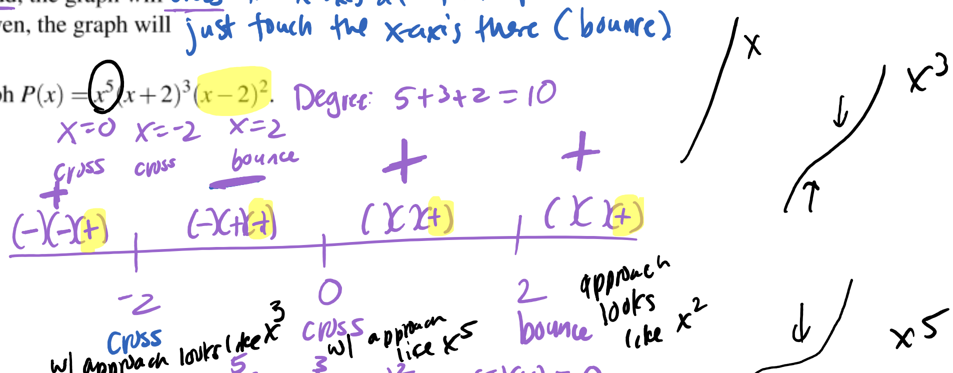

NOTE: In the previous example, the graph did not cross the $x$−axis at all zeros. If $x - c$ occurs $m$ times in a factorization, we say $c$ is a zero with multiplicity $m$.

If $m$ is odd, the graph will cross the x-axis at that point.

If $m$ is even, the graph will just touch the x-axis there (bounce)

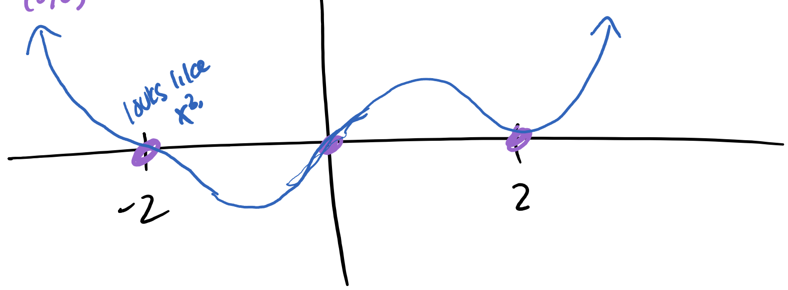
Ex: Graph $P(x) = x^5(x+2)^3(x-2)^2$. Degree: $5+3+2=10$

zeros: $x=0$ cross, $x=-2$ cross, $x=2$ bounce




y-int: Set $x=0$: $P(0) = 0^5(0+2)^3(0-2)^2 = 0(8)(4) = 0$

y-int: $(0,0)$



$x = -3$ is zero $\Rightarrow$ factor is $(x+3) = 0$



Ex: Find an equation of a polynomial with:

- degree 5
- Roots of multiplicity 2 at $x = -3$ and $x = 2$
- Root of multiplicity 1 at $x = -2$
- $y$-intercept at $(0, 4)$

factor $(x+3)^2$ factor $(x-2)^2$ factor $(x+2)^1$

$f(x) = (x+3)^2(x-2)^2(x+2)$

~~Check this w/ y-int: $x=0$, $y=4$~~

~~$f(0) = (0+3)^2(0-2)^2(0+2) \stackrel{?}{=} 4$~~

~~$9(4)(2) \neq 4$~~

$f(x) = a(x+3)^2(x-2)^2(x+2)$

Use y-int again; solve for a this time

$(0,4)$

$f(0) = a(3)^2(-2)^2(2) = 4$

$9(\cancel{4})(2)a = \cancel{4}$

$18a = 1$

$a = \frac{1}{18}$

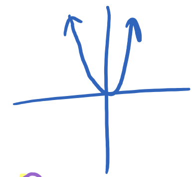
$$\boxed{f(x) = \frac{1}{18}(x+3)^2(x-2)^2(x+2)}$$

$\frac{72a}{72} = \frac{4}{72}$

$a = \frac{2}{36} = \frac{1}{18}$

Scratch:

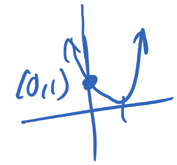
$y = x^2 = (x-0)^2$



$y = 1(x-1)^2$

zero: $x = 1$

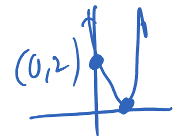
degree: 2




$y = 2(x-1)^2$

zero: $x = 1$

degree: 2




$y = \frac{1}{2}(x-1)^2$

zero: $x = 1$

degree: 2