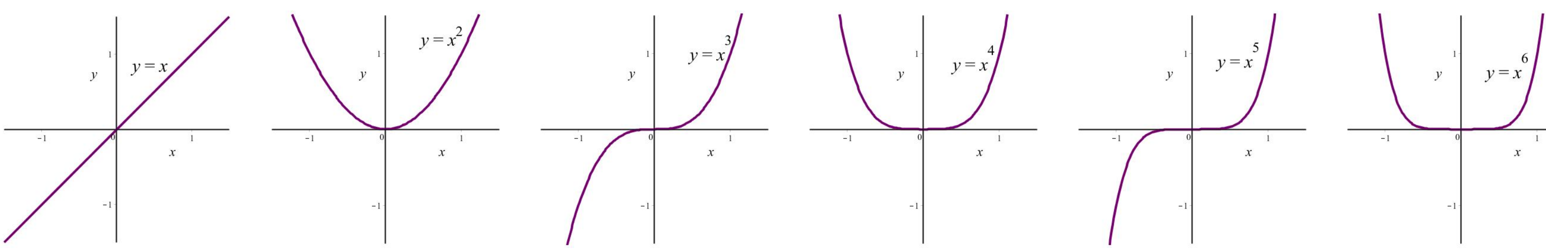
Math 1314 – College Algebra
Sections 5.2-5.3 Power Functions and Polynomial Functions/Graphs of Polynomial Functions

- A polynomial function in one variable (let's use x) is $f(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \ldots + a_1 x + a_0$.
  ■ Degree is n, highest power of x ■ Leading term is $a_n x^n$ ■ Leading coefficient is a_n ■ Constant term is a_0
- Looks like: Sum and/or difference of terms whose variable x has whole number powers. Coefficients are real numbers.
- Examples of polynomials:

Function	Example	Degree	Graph
Constant Fcn	$f(x) = 5$	0	
Linear Fcn	$g(x) = 3x + 11$	1	
Quadratic Fcn	$h(x) = 2x^2 - 17x + 5$	2	

- If $f(x) = a_0$ where $a_0 \neq 0$, f has degree 0. If $f(x) = 0$, f has no degree.
- Graphs of polynomials are always smooth and continuous.

Graphs of Monomials:



- x^n has the same general shape as _____ when n is even and the same general shape as _____ when n is odd.
- End behavior: What happens to y-values as x becomes large $+$ and large $-$
  - $x \to \infty$: large in the positive direction
  - $x \to -\infty$: large in the negative direction
- End behavior is determined by the term with the highest power of x.
  - For x^{even}, tails point in
  - For x^{odd}, tails point in
- Given $P(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \ldots + a_1 x + a_0$,

P has odd degree:		P has even degree:	
Leading Coefficient $+$:	Leading Coefficient $-$:	Leading Coefficient $+$:	Leading Coefficient $-$:

Ex: Find the end behavior:

(a) $f(x) = 3x^4 + 4x^3 + 5x^2 + 6x + 7$

(b) $g(x) = -19x^{15} - 2x^4 + 3x^3 + 6x - 2$

(c) $h(x) = -8x^6 + 3x^4 - 2x^3 - 17$

Intermediate Value Theorem for Polynomials: If P is a polynomial function and $P(a)$ and $P(b)$ have opposite signs, then there exists at least one value c between a and b such that
(This will be proven in Calculus).

IMPORTANT: Between any two successive zeros, the values of a polynomial are

(Between any 2 successive zeros, the graph of a polynomial lies entirely ______________ or ______________ the ______________

Ex: Graph $y = -x^4 + 10x^2 - 9 = -1(x-3)(x+3)(x-1)(x+1)$. Consider symmetry.

Ex: Let $P(x) = x^3 - x^2 - x + 1$. Find zeros and sketch.

NOTE: In the previous example, the graph did not cross the $x-$axis at all zeros. If $x - c$ occurs m times in a factorization, we say c is a zero with
If m is odd, the graph will
If m is even, the graph will

Ex: Graph $P(x) = x^5(x+2)^3(x-2)^2$.

Ex: Find an equation of a polynomial with:

- degree 5
- Roots of multiplicity 2 at $x = -3$ and $x = 2$
- Root of multiplicity 1 at $x = -2$
- y-intercept at $(0, 4)$