Math 1314 – College Algebra
Section 5.4 Dividing Polynomials (including some of Section 5.5)

- Polynomial equation: $P(x) = 0$ where $P(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \ldots + +a_2x^2 + a_1x + a_0$. n is a natural number and gives the degree of the polynomial.

- Zero of a polynomial $P(x)$: is any number r for which $P(r)=0$

Ex: Use long division to simplify $\frac{9x^4 + 8x^2 - 3x + 4}{3x^2 - x + 1}$.

$$3x^2 + x + 2 + \frac{-2x+2}{3x^2-x+1}$$

$$\begin{array}{r|l} & 9x^4 + 0x^3 + 8x^2 - 3x + 4 \\ 3x^2-x+1 & -9x^4 \pm 3x^3 \mp 3x^2 \\ & 3x^3 + 5x^2 - 3x + 4 \\ & -3x^3 \pm x^2 \mp x \\ & 6x^2 - 4x + 4 \\ & -6x^2 \pm 2x \mp 2 \\ & -2x + 2 \end{array}$$

$\frac{317}{5} \Rightarrow$

$$\begin{array}{r|l} & 63\frac{2}{5} \\ 5 & 317 \\ & -30 \\ & 17 \\ & -15 \\ & 2 \end{array}$$

$60 + 3$

$\frac{317}{5} = 63R2$

$63 + \frac{2}{5}$

$\frac{4}{10}$

$\frac{317}{5} = 63 + \frac{2}{5}$

63.4

$$\frac{9x^4 + 8x^2 - 3x + 4}{3x^2-x+1} = (3x^2+x+2) + \frac{(-2x+2)}{(3x^2-x+1)}$$

$$\Rightarrow \underbrace{9x^4 + 8x^2 - 3x + 4}_{\text{Dividend}} = \underbrace{(3x^2+x+2)}_{\text{Quotient}}\underbrace{(3x^2-x+1)}_{\text{Divisor}} + \underbrace{(-2x+2)}_{\text{Remainder}}$$

- $\underbrace{P(x)}_{\text{Dividend}} = \underbrace{(x-r)}_{\text{Divisor}}\underbrace{Q(x)}_{\text{Quotient}} + \underbrace{R(x)}_{\text{Remainder}}$

- Synthetic Division – We can divide polynomials by linear terms using synthetic division. Divide $P(x)$ by $x - c$ or $x + c$.

Ex: Use synthetic division to simplify $\frac{3x^3 - 12x^2 - 9x + 1}{x - 5}$.

$$\begin{array}{r|rrr|r} 5 & 3 & -12 & -9 & 1 \\ & & 15 & 15 & 30 \\ \hline & 3 & 3 & 6 & 31 \end{array}$$

$$\begin{array}{r|l} & 3x^2 + 3x + 6 \\ x-5 & 3x^3 - 12x^2 - 9x + 1 \\ & -3x^3 \pm 15x^2 \\ & 3x^2 - 9x + 1 \\ & -3x^2 \pm 15x \\ & 6x + 1 \\ & -6x \pm 30 \\ & 31 \end{array}$$

$\frac{x^3}{x} = x^2$

$$\underbrace{3x^3 - 12x^2 - 9x + 1}_{\text{Dividend}} = \underbrace{(x-5)}_{\text{Divisor}}\underbrace{(3x^2+3x+6)}_{\text{Quotient}} + \underbrace{31}_{\text{Remainder}}$$

$$\begin{array}{r} 3x^3 + 3x^2 + 6x \\ -15x^2 - 15x - 30 \\ + 31 \\ \hline 3x^3 - 12x^2 - 9x + 1 \end{array}$$

$f(x) = 3(x-2)(x+3) \leftarrow$ factorization

Zeroes: $x=2$, $x=-3$

- Factor Theorem: If $P(x)$ is a polynomial and r is a zero of $P(x)$ (i.e. $P(r)=0$) then $x-r$ is a factor.
  Also, if $x-r$ is a factor, then $P(r)=$ 0 (r is a zero of $P(x)$).

- Remainder Theorem: If $P(x)$ is divided by $(x-r)$ the remainder is $P(r)$

Real Zeros of Polynomials: If P is a polynomial and c is a real number, then the following are equivalent:

1. c is a zero of P.
2. $x=c$ is a soln to eqn $P(x)=0$
3. $x-c$ is a factor of $P(x)$
4. $(c,0)$ is x-intercept of $P(x)$

Ex: Find $P(5)$ for $P(x) = 3x^3 - 12x^2 - 9x + 1$.

$$P(5) = 3(5)^3 - 12(5)^2 - 9(5) + 1$$
$$= 3(125) - 12(25) - 45 + 1$$
$$= 375 - 240 - 60 - 45 + 1$$
$$= 31$$

$3(125) = 3(100 + 25)$

$300 + 75$

$12(25) = 12(20) + 12(5)$

Ex: Let $P(x) = 2x^3 + 7x^2 + 6x - 5$. Show that $x - \frac{1}{2}$ is a factor of $P(x)$.

$\frac{1}{2}$	2	7	6	-5
		1	4	5
	2	8	10	0

depressed poly.

$$P(x) = \left(x - \tfrac{1}{2}\right)(2x^2 + 8x + 10)$$
$$= \left(x - \tfrac{1}{2}\right)(2)(x^2 + 4x + 5) = 2\left(x - \tfrac{1}{2}\right)(x^2 + 4x + 5)$$
$$P(x) = (2x - 1)(x^2 + 4x + 5)$$

Ex: Find a polynomial of degree four with zeroes $7, -7, 2, 3$.

Factors: $(x-7)(x+7)(x-2)(x-3)$

$$P(x) = a(x-7)(x+7)(x-2)(x-3)$$

there are infinitely many $\Rightarrow$ can choose any value for a.