Math 1314 – College Algebra
Section 5.6 Rational Functions

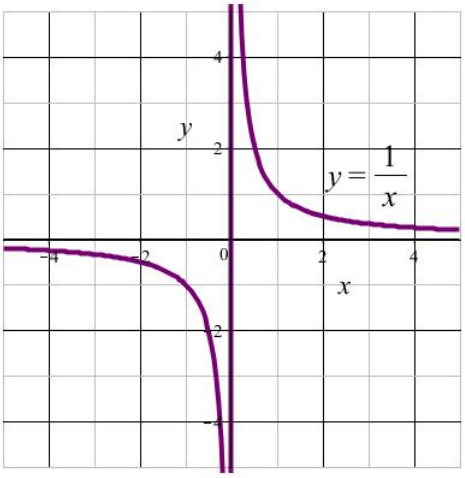
- A rational function is a function of the form $y = \dfrac{P(x)}{Q(x)}$ where $P(x)$ and $Q(x)$ are polynomials.
- Always check to see if we can factor the numerator and denominator. If there are any terms in common in the numerator and denominator, we will have a hole in the graph.
- Vertical Asymptote: The line $x = a$ is a VA of the graph of $y = f(x)$ if the y values of the function approach ∞ or $-\infty$ as x approaches a.
- A good place to start looking for VA is where the function is not defined. NOTE: Not all values of x where the function is not defined will be VAs.
- Horizontal Asymptote: The line $y = b$ is a HA of the graph of $y = f(x)$ if the y values of the function approach b as x gets very large and positive (approaches ∞) or as x gets very large and negative (approaches $-\infty$).
- To find horizontal asymptotes – Use long division or divide by the highest power of x in the denominator.
  Recall: As $x \to \infty$, $\dfrac{\#}{x^n} \to 0$ if n is a positive integer. As $x \to -\infty$, $\dfrac{\#}{x^n} \to 0$ if n is a positive integer.

  Ex: Sketch $f(x) = \dfrac{1}{x}$. Find the domain, range, and any asymptotes.
  Domain: Set denom $\neq 0$: $x \neq 0$ so VA at $x = 0$

  As $x \to -\infty$, $\dfrac{1}{x} \to 0$ (below the x-axis)

  As $x \to \infty$, $\dfrac{1}{x} \to 0$ (above the x-axis)

x	y
-3	$-\frac{1}{3}$
-2	$-\frac{1}{2}$
-1	-1
$-\frac{1}{2}$	-2
$-\frac{1}{3}$	-3
1	1
$\frac{1}{3}$	3
$\frac{1}{2}$	2
2	$\frac{1}{2}$
3	$\frac{1}{3}$



- To find HA of a rational function $y = \dfrac{P(x)}{Q(x)}$:
  - If degree of numerator $<$ degree of denominator, HA at $y = 0$ (x-axis)
  - If degree of numerator $>$ degree of denominator, no HA. (may have slant or oblique asymptote)
  - If degree of numerator $=$ degree of denominator, HA at $y = \dfrac{\text{lead coefficient of numerator}}{\text{lead coefficient of denominator}}$

Ex: Find all asymptotes:
(a) $f(x) = \dfrac{4x+2}{3x-3}$

(b) $g(x) = \dfrac{2x+3}{x^2-5x-14}$

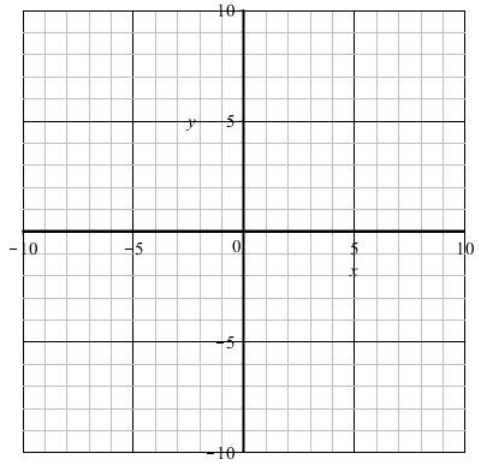
(c) $h(x) = \dfrac{x^2+5x}{x-1}$

- Graphing Strategy: Try to factor the numerator and denominator. Find all intercepts (set numerator= 0 for x-intercepts), symmetry, vertical asymptotes (set denominator = 0), and horizontal asymptotes. Plot points.

Ex: Graph $f(x) = \dfrac{3x}{x-2}$.

Factor numerator and denominator. Are there any terms in common?

x	y



Set numerator= 0:

Set denominator= 0:

Thus, x-intercepts are:

Thus, VA are:

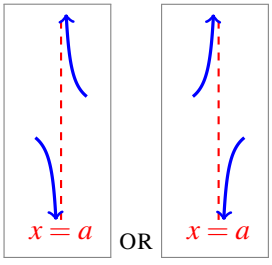
Find y-intercept:

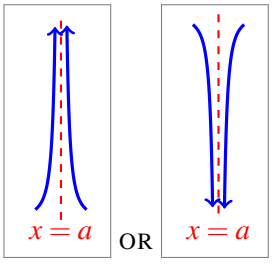
Find HA:

Solve HA=function to see if graph crosses its HA.

- Regarding the multiplicity of factors in the denominator:

If multiplicity is ODD:	If multiplicity is EVEN:
The graph will have opposite behavior to left and right of the VA where that factor=0. One tail will run to ∞ and the other will run to $-\infty$.	The graph will have same behavior to left and right of the VA where that factor=0. Both tails will run to ∞ or both will run to $-\infty$.



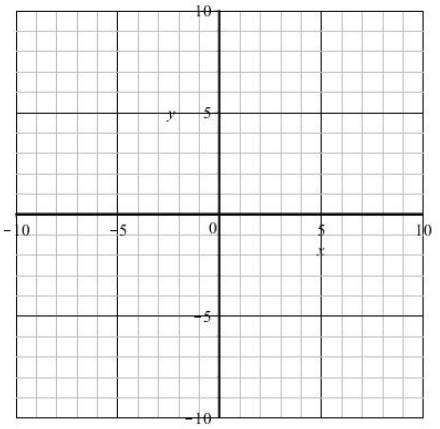


Ex: Graph $f(x) = \dfrac{3x(x+7)}{(x+7)(x-2)}$

HOLE:

VA:

HA:



- If a function $y = f(x)$ has a vertical asymptote at $x = a$, the graph of $y = f(x)$ will never cross the vertical line $x = a$.
- If a function $y = f(x)$ has a horizontal asymptote at $y = b$, the graph of $y = f(x)$ may cross the horizontal line $y = b$. HA just tells us how the function behaves in the long run.

Ex: Graph $g(x) = \dfrac{3x+6}{x^3-x^2-5x-3} = \dfrac{3x+6}{(x+1)^2(x-3)}$ (Give the basic shape).

Factor numerator and denominator. Are there any terms in common?

x	y

Set numerator= 0:

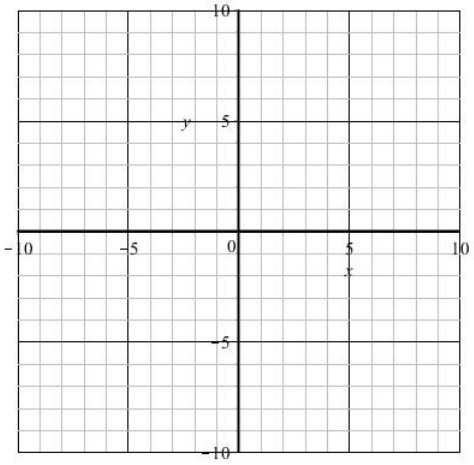
Set denominator= 0:

Thus, x-intercepts are:

Thus, VA are:

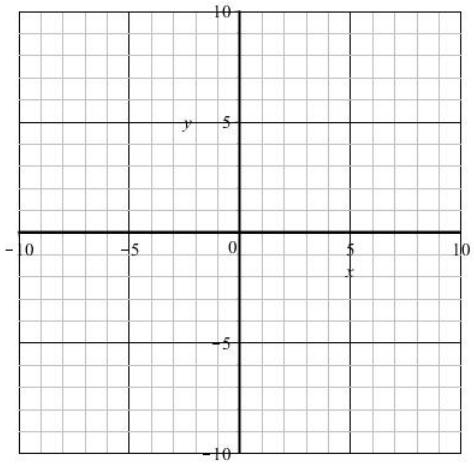
Find y-intercept:

Find HA:

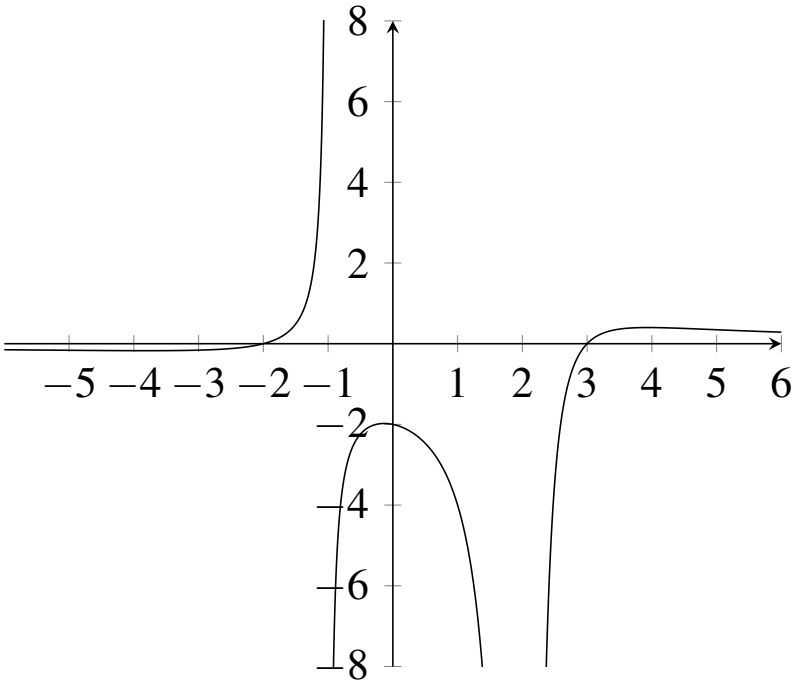


Solve HA=function to see if graph crosses its HA.

Ex: Find the domain and graph $f(x) = \dfrac{x^2 - x - 12}{x - 4} = \dfrac{(x-4)(x+3)}{(x-4)}$



Ex: Find the rational function given the graph below. Use least degree.



For you to do:

Sketch $f(x) = \dfrac{1}{x-3}$ and $f^{-1}(x) = \dfrac{3x+1}{x}$ on the same graph. Show all vertical and horizontal asymptotes.