

Math 1314 – College Algebra
Section 6.1-6.2 Exponential Functions/Graphs of Exponential Functions

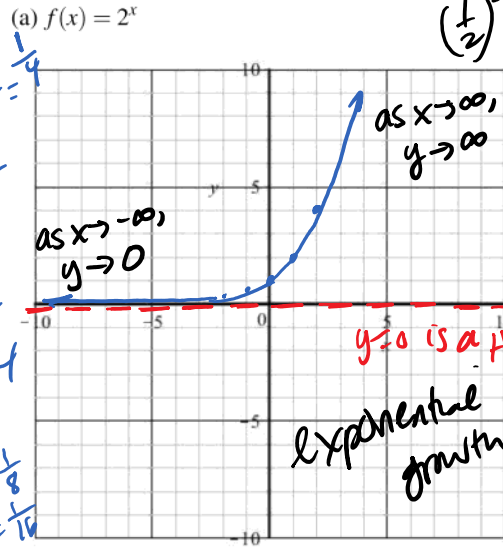
■ Exponential functions look like $y = b^x$ or $f(x) = b^x$ where $b > 0, b \neq 1$ and x is a real number.

■ Why can b not be 1?

$x^{-n} = \frac{1}{x^n}$ $\frac{1}{x^{-n}} = x^n$ $1^x = 1$ $y = 1$

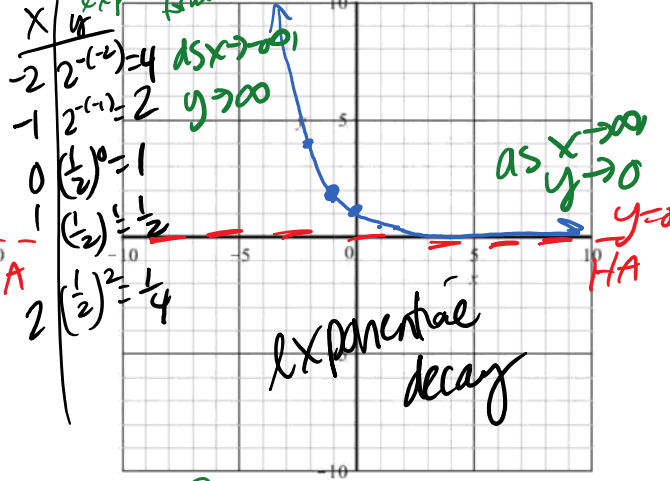
Ex: (a) Graph $f(x) = 2^x$ and $g(x) = \left(\frac{1}{2}\right)^x$. Give the domain, range, and end behavior of both.

x	y
-2	$2^{-2} = \frac{1}{2^2} = \frac{1}{4}$
-1	$2^{-1} = \frac{1}{2}$
0	$2^0 = 1$
1	$2^1 = 2$
2	$2^2 = 4$
-3	$2^{-3} = \frac{1}{8}$
-4	$2^{-4} = \frac{1}{16}$
-5	$\frac{1}{32}$
-6	$\frac{1}{64}$

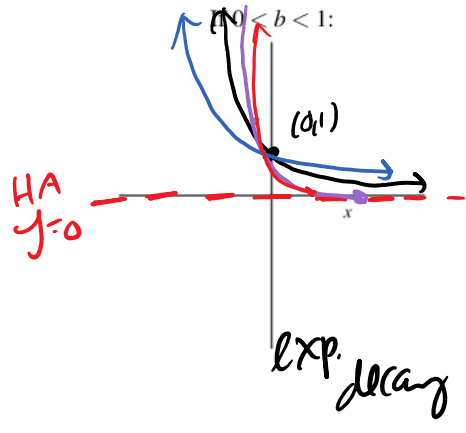
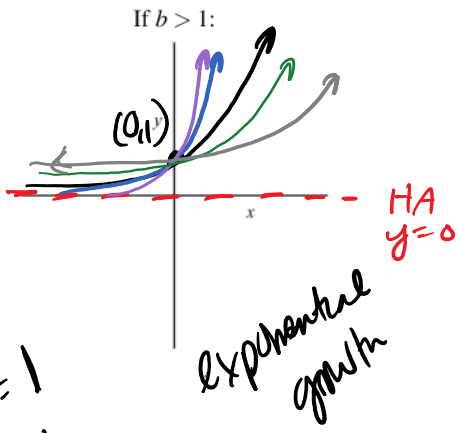


Domain: \mathbb{R}
 Range: $(0, \infty)$
 End behavior:
 As $x \rightarrow \infty,$
 As $x \rightarrow -\infty,$ See graph

$\left(\frac{1}{2}\right)^{-2} = 2^2$ (b) $g(x) = \left(\frac{1}{2}\right)^x = \frac{1^x}{2^x} = \frac{1}{2^x} = 2^{-x}$ (reflect $y=2^x$ @ y-axis)



Domain: \mathbb{R}
 Range: $(0, \infty)$
 End behavior:
 As $x \rightarrow \infty,$
 As $x \rightarrow -\infty,$ See graph

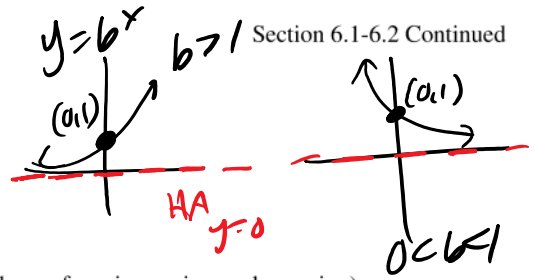


$f(x) = b^x$
 $f(0) = y = b^0 = 1$
 $b > 0$
 $b \neq 1$

$f(1) = b^1 = b$ Math 1314

■ Properties of Exponential Functions $f(x) = b^x$:

- Domain: \mathbb{R}
- Range: $(0, \infty)$
- y-intercept: $(0, 1)$
- Horizontal asymptote at $y = 0$
- Passes through the point $(1, b)$
- If $b > 1$, increasing
- If $0 < b < 1$, decreasing



(Choose from increasing or decreasing)
(Choose from increasing or decreasing)

Ex: Find the formula for an exponential function $f(x) = ab^x$ that passes through the points $(-1, \frac{7}{2})$ and $(3, 56)$. Find a (stretch factor) & b (base)

$$\frac{7}{2} = ab^{-1}$$

$$56 = ab^3$$

$$\frac{7}{2} = \frac{a}{b}$$

Solve for a

$$\frac{7b}{2} = a$$

$$56 = \left(\frac{7b}{2}\right)\left(\frac{b^3}{1}\right)$$

$$56 = \frac{7b^4}{2}$$

$$112 = 7b^4$$

$$\frac{112}{7} = b^4$$

$$16 = b^4$$

$$b = 2$$

$$\frac{7(2)}{2} = a$$

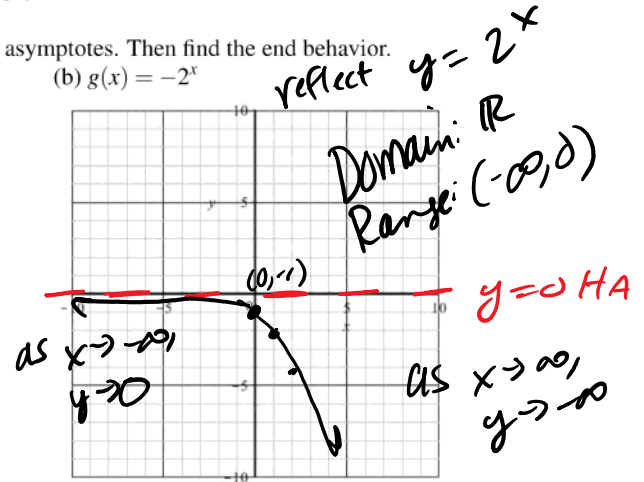
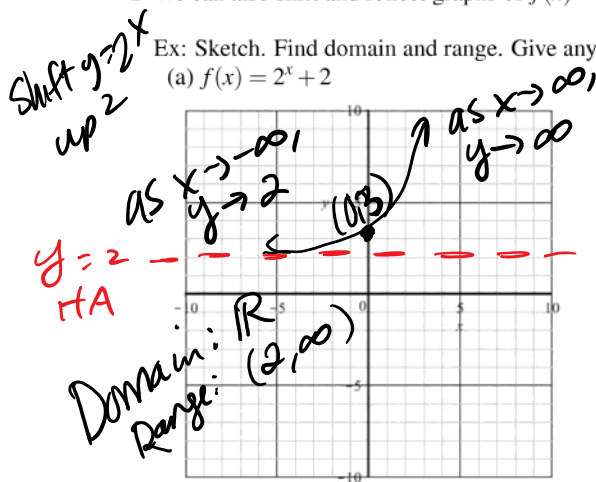
$$a = 7$$

$$f(x) = ab^x$$

$$f(x) = 7(2^x)$$

■ We can also shift and reflect graphs of $f(x) = b^x$.

Ex: Sketch. Find domain and range. Give any asymptotes. Then find the end behavior.

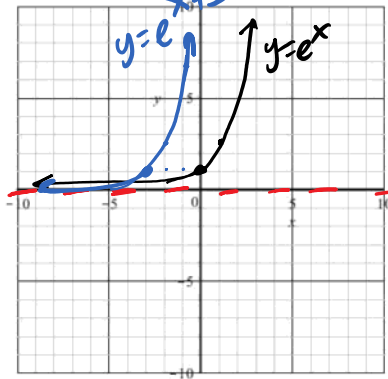


- $e \approx 2.71812818284\dots$

Ex: Sketch $f(x) = e^x$

Then write a function that would shift $f(x)$ 3 units to the left. Graph.

b > 1 growth:
 $\begin{array}{r|l} x & y \\ \hline 0 & 1 \\ 1 & b=e \end{array}$



$$y = e^{x+3}$$

b > 0 b ≠ 1

- Compound interest Formula: If P dollars are deposited in an account earning interest at an annual rate r , compounded n times each year, the amount A in the account after t years is given by $A = P \left(1 + \frac{r}{n}\right)^{nt}$

Ex: You want to save up enough money to buy a herd of llamas. It's a big herd so you need \$10,000. If you deposit \$7800 into an account paying 6.2% interest compounded monthly, will you have enough money to buy the herd in 4 years?

PEMDAS

$A = ?$
 $P = 7800$
 $r = .062$
 $n = 12$
 $t = 4$

$$A = P \left(1 + \frac{r}{n}\right)^{nt}$$

$$A = 7800 \left(1 + \frac{.062}{12}\right)^{12(4)}$$

$A = \$9989.01$
 ☹️ Not enough to buy llamas

Calculator input: .062 [÷] 12 [=] + 1 [=] [y^x] [12] [×] 4 [=] [×] 7800 [=]

- Radioactive decay: The atomic structure of a radioactive material changes as the material emits radiation. The half life of such a material is the time it takes for half of the sample to decompose.

Ex: Suppose we have 100 grams of substance X, which has a half-life of 10 years. Suppose further that substance X is safe at levels of 1 gram or less. How long will it take for substance X to decay to a safe level?

time (yrs)	t=0	t=10	t=20	t=30	t=40	t=50	t=60	t=70
Amount	100g	50g	25g	12.5g	6.25g	3.125g	1.5625g	0.78125g

Substance X is at a safe level somewhere b/w 60 & 70 yrs.

