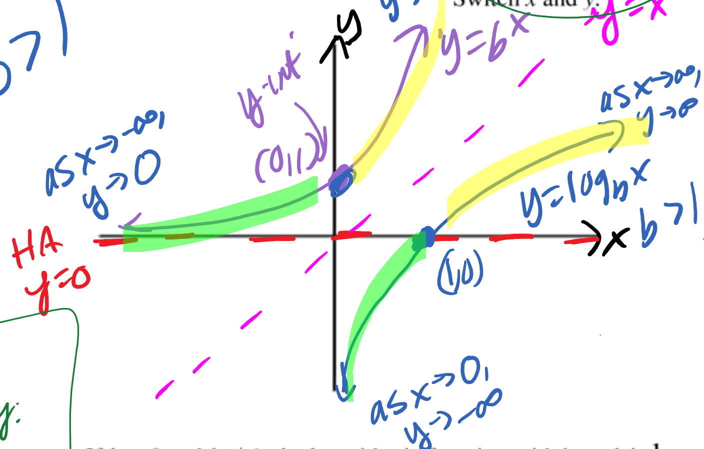
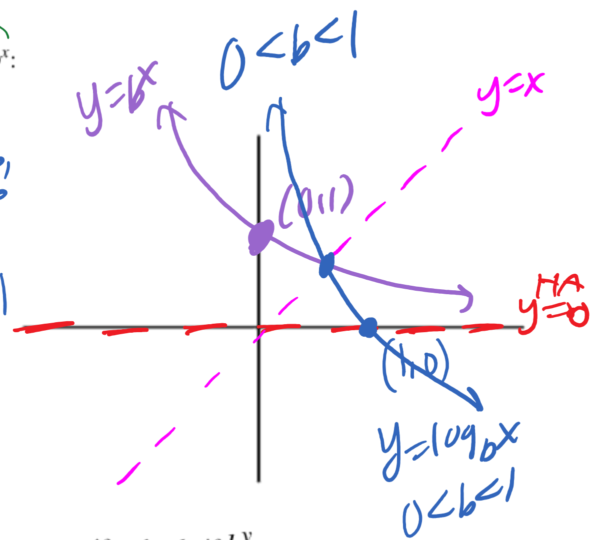
# Math 1314 – College Algebra
## Section 6.3-6.4 Logarithmic Functions/Graphs of Logarithmic Functions

- Are exponential functions one-to-one? YES!

Find the inverse of $y = b^x$: Switch $x$ and $y$: $y=x$

$b>1$





$y=b^x$
Switch $x$ & $y$:
$x=b^y$

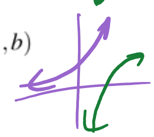
- If $b>0$ and $b \neq 1$, the logarithmic function with base $b$ is $\log_b x = y$ if and only if $b^y = x$

$$\log_b \underset{\text{input}}{m} = \underset{\text{output}}{n} \Longleftrightarrow b^{n} = m$$

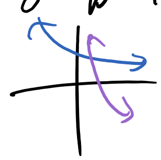
input, output, Inverse

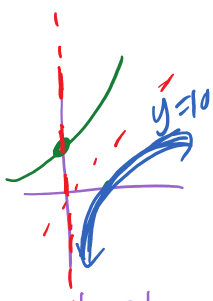
| Properties of Exponential Functions $f(x)=b^x$: | Properties of Logarithmic Functions $f(x)=\log_b x$: |
|---|---|
| ▪ Domain: $(-\infty,\infty)$ | ▪ Domain: $(0,\infty)$ |
| ▪ Range: $(0,\infty)$ | ▪ Range: $(-\infty,\infty)$ |
| ▪ $y$-intercept: $(0,1)$ | ▪ $x$-intercept: $(1,0)$ |
| ▪ Horizontal asymptote at $y=0$ | ▪ Vertical asymptote at $x=0$ |
| ▪ Passes through the point $(1,b)$ | ▪ Passes through the point $(b,1)$ |
| ▪ If $b>1$, increasing | ▪ If $b>1$, increasing |
| ▪ If $0<b<1$, decreasing | ▪ If $0<b<1$, decreasing |

$b>1$



$0<b<1$



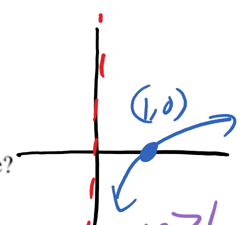


$y=\log_b x$

$b>1$

Ex: Find (a) $\log_{10}(-13)$ X

Domain: $(0,\infty)$



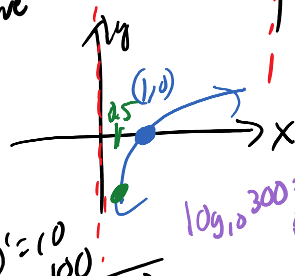
$b>1$

(b) $\log_{10}(0)$ X

Domain: $(0,\infty)$

Ex: What sign will $\log_{10}(.5)$ have? negative

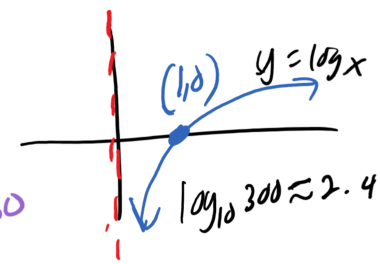
$\log_{10}(.5) \approx -0.3010$



$10^1=10$
$10^2=100$
$10^3=1000$

$\log_{10} 300 = c$
$\Updownarrow$
$10^c = 300$

What sign will $\log_{10}(300)$ have? positive



$\log_{10} 300 \approx 2.4771$

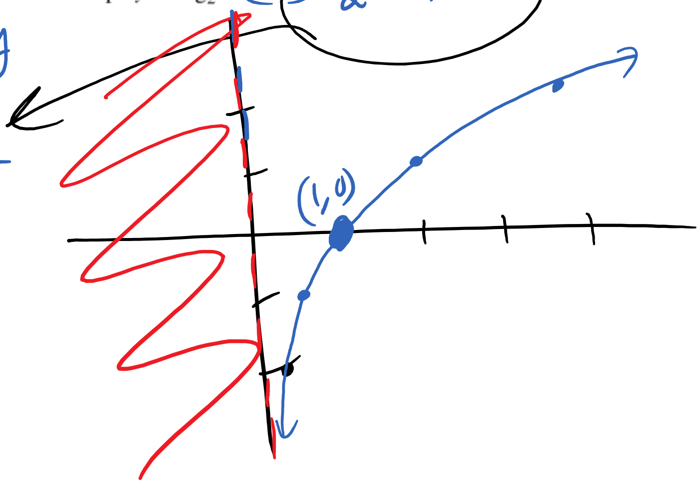
$\log_b m = n \iff b^n = m$

Ex: Graph $y = \log_2 x$. $\iff 2^y = x$ (power is $y$)

Domain: $(0, \infty)$

$\log_2 x = y$

| $x$ | $y$ |
|---|---|
| $2^{-2} = \frac{1}{4}$ | $-2$ |
| $2^{-1} = \frac{1}{2}$ | $-1$ |
| $2^0 = 1$ | $0$ |
| $2^1 = 2$ | $1$ |
| $2^2 = 4$ | $2$ |




Ex: Find m:

$\log_b m = n \iff b^n = m$

(a) $\log_3 9 = m$ $\iff 3^m = 9$

$m = 2$

$3^0 = 1$, $3^1 = 3$, $3^2 = 9$, $3^3 = 27$

$\log_b m = n \iff b^n = m$

(b) $\log_8 \frac{1}{64} = m$ $\iff 8^m = \frac{1}{64}$

$8^m = \frac{1}{64} \Rightarrow 8^m = (64^{-1})$

$m = -2$

$\log_b m = n \iff b^n = m$

(c) $\log_4 1 = m$ $\iff 4^m = 1$

$m = 0$

(d) $\log_m \frac{1}{16} = -2$ $\iff m^{-2} = \frac{1}{16}$

$\frac{1}{m^2} = \frac{1}{16}$

$m = 4$

$\log_b m = n \iff b^n = m$

$\log_b m = n \Leftrightarrow b^n = m$

(e) $\log_m 3 = \frac{1}{2}$ $\iff m^{\frac{1}{2}} = 3$

$m = 9$

(f) $\log_5 m = 2$ $\iff 5^2 = m$

$m = 25$

- Base 10 logarithms: The logarithm with base 10 is the common logarithm. $\log x$ means $\log_{10} x$.

- $\log_{10} \frac{1}{100} = -2$ because $10^{-2} = \frac{1}{100}$
  $\log \frac{1}{10} = -1$ because $10^{-1} = \frac{1}{10}$
  $\log 1 = 0$ because $10^0 = 1$
  $\log 10 = 1$ because $10^1 = 10$
  $\log 100 = 2$ because $10^2 = 100$
  $\log 1000 = 3$ because $10^3 = 1000$

$\log_b m = n \Leftrightarrow b^n = m$



Ex: Find $x$ to four decimal places: $\log_{10} x = 0.7482$ $\Leftrightarrow$ $10^{.7482} = x$

$x = 5.6002$

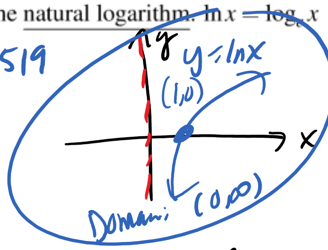
log

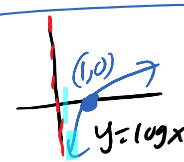
$e \approx 2.718128\ldots$ base $= e \approx 2.718\ldots$

- The logarithm with base $e$ is the natural logarithm: $\ln x = \log_e x$

Ex: Find (a) $\ln 17.32 \approx 2.8519$ (b) $\ln(\log 0.05)$ $\ln(-\#)$ X

$e^{2.8519}$




$\log_b m = n \Leftrightarrow b^n = m$

operand

Ex: Solve (a) $\ln_e x = 1.335$ (b) $\ln_e x = \log 5.5$

$e^{1.335} = x$ $e^{\log 5.5} = x$

$x = 3.8000$ $x \approx 2.0967$

$(f \circ f^{-1})(x) = x$

$(f \circ f^{-1})(\star) = \star$

$e^{\ln e \star}$

| |
|---|
| NOTE: |
| ■ $e^{(\ln(\star))} = \star$ ■ $10^{(\log(\star))} = \star$ ■ $\ln e^{(\star)} = \star$ ■ $\log 10^{(\star)} = \star$ |

$e^{\ln 2}$

We will discuss this in more detail in the next section. $\log x$ $10^x$

Ex: Simplify: (a) $e^{(\ln_e(17.5))} - 11$ (b) $\log_{10} 10^{(13)} + 6$

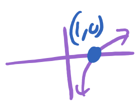
$= 17.5 - 11$ $\log 10^{13}$ $= 13 + 6$

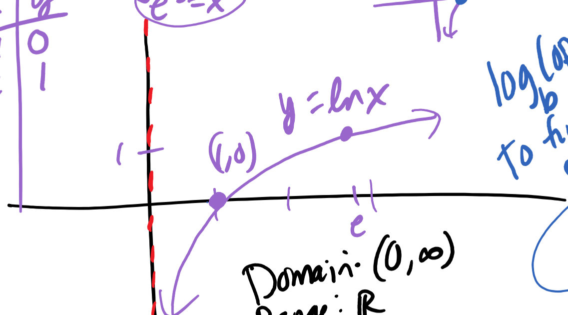
$= 6.5$ $= 19$

Ex: Graph and find the domain:

(a) $y = \ln \boxed{x}$ base $= e > 1$ $e^y = x$ (b) $y = \log x$ base $= 10$ passes through $(b,1)$

| x | y |
|---|---|
| 1 | 0 |
| $e^1 = e$ | 1 |

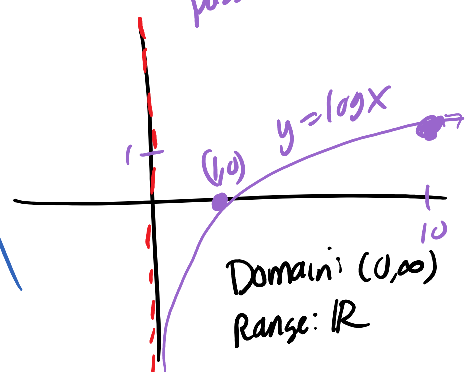





$\log_b(\text{operand})$ To find domain, solve operand $> 0$

Domain: $(0, \infty)$
Range: $\mathbb{R}$

VA $x = 0$ To find VA, solve operand $= 0$




Domain: $(0, \infty)$
Range: $\mathbb{R}$
VA $x = 0$

- We can also shift and reflect graphs of $f(x) = \log_b x$.

Ex: Graph $f(x) = \log_2(x+2)+5$. Find the domain, range, and vertical asymptote.

Shift $y=\log_2 x$ left 2 units. Then up 5 units

Parent fcn: $y=\log_2 x$

$2^y = x$

| x | y |
|---|---|
| 1 | 0 |
| 2 | 1 |

Domain: operand $>0$

$x+2>0$

$x>-2$

$(-2,\infty)$

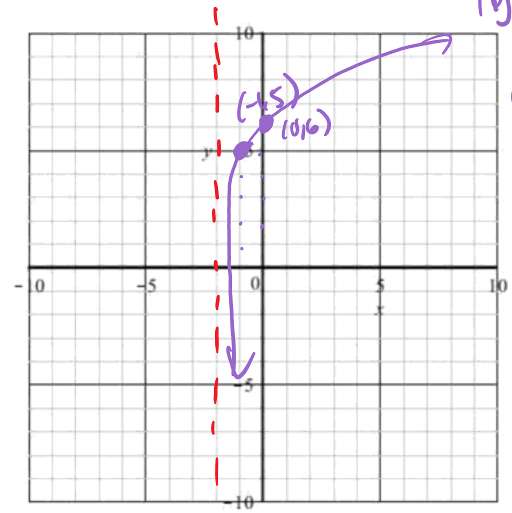
Range: $\mathbb{R}$

VA at $x=-2$

operand $=0$

$x+2=0$

$x=-2$




Ex: Find the domain, range, and vertical asymptote of $g(x) = \log(12-2x)$

Domain: Solve operand $>0$

$12-2x>0$

$12>2x$

$6>x$

$x<6$

$(-\infty,6)$

Range: $\mathbb{R}$

VA: $x=6$

6

Ex: Find the domain of $(f+g)(x) = f(x)+g(x)$

$(f+g)(x) = \log_2(x+2)+5 + \log(12-2x)$

operands $>0$

$x+2>0$

$x>-2$

AND

$12-2x>0$

$x<6$

$x>-2$ AND $x<6$

-2 6

-2 6

Domain: -2 6

Domain: $(-2,6)$

Ex: Given $f(x) = \log_2(x+2)+5$, find its inverse. Then graph both.

$y = \log_2(x+2)+5$

Switch x & y:

$x = \log_2(y+2)+5$ Solve for y

$(x-5) = \log_2(y+2)$

$2^{(x-5)} = y+2$

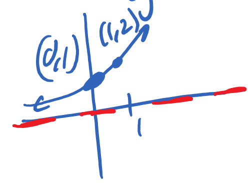
$2^{x-5} - 2 = y$

$f^{-1}(x) = 2^{x-5} - 2$

$\log_b m = n \Leftrightarrow b^n = m$

$\log_b m = n \Leftrightarrow b^n = m$

Parent fcn $y = 2^x$




Shift right 5
down 2