Math 1314 – College Algebra
Section 6.5 Logarithmic Properties

power

- Recall: $\log_b m = n \iff b^n = m$

$\log_b m = n \Updownarrow b^n = m$

Laws of Exponents: Let a and b be positive numbers and let x and y be real numbers. Then,

- $b^x b^y = b^{x+y}$ ■ $\frac{b^x}{b^y} = b^{x-y}$ ■ $(b^x)^y = b^{xy}$ ■ $(ab)^x = a^x b^x$ ■ $\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}$

Let $f(x) = b^x$ $f^{-1}(x) = \log_b x$

Properties of Logarithms: If b, M, N are all positive numbers and $b \neq 1$, then $x = (f \circ f^{-1})(x) = f(f^{-1}(x)) = f(\log_b x)$

- $\log_b 1 = 0 \iff b^0 = 1$ ■ $\log_b b = 1 \iff b^1 = b$ ■ $\log_b b^x = x$ ■ $b^{\log_b x} = x$
- $\log_b(MN) = \log_b M + \log_b N$ ■ $\log_b\left(\frac{M}{N}\right) = \log_b M - \log_b N$

$x = f^{-1}(f(x)) = f^{-1}(b^x)$ $= \log_b b^x$

$\log_b(M^2) = \log_b(MM) = \log_b M + \log_b M = 2\log_b M$

- $\log_b M^p = p \log_b M$ ■ If $\log_b x = \log_b y$, then $x = y$.

- Recall: $y = b^x$ and $y = \log_b x$ are inverses of each other.
Also, $(f \circ f^{-1})(x) = x$ and $(f^{-1} \circ f)(x) = x$ (property of inverses).

$x = (f \circ f^{-1})(x) = f(f^{-1}(x)) = f(\log_b x) = b^{\log_b x}$
$x = (f^{-1} \circ f)(x) = f^{-1}(f(x)) = f^{-1}(b^x) = \log_b b^x$

$(\sqrt{xy})^2 = xy$

Ex: Simplify

(a) $\log_3 1 = m \iff 3^m = 1$, $m = 0$ (b) $\log_2 2 = m \iff 2^m = 2$, $m = 1$ (c) $7^{\log_7 13} = 13$ (d) $\log_4 4^5 = 5$

$\log_b(MN) = \log_b M + \log_b N$ $\log_b\left(\frac{M}{N}\right) = \log_b M - \log_b N$

Ex: Assume x, y, z, b are positive numbers and $b \neq 1$. Expand

(a) $\log_b(xyz) = \log_b x + \log_b y + \log_b z$ (b) $\log_b\left(\frac{xy}{z}\right) = \log_b x + \log_b y - \log_b z$

NOTE: If an expression is in the numerator of the coefficient of a logarithm in condensed form, the coefficient of the logarithm of that expression will be _+__ when the logarithm is expanded.
If an expression is in the denominator of the coefficient of a logarithm in condensed form, the coefficient of the logarithm of that expression will be _−__ when the logarithm is expanded.

(c) $\log_b\left(\frac{3x^2}{4yz}\right)$

$\log_b(3) + \log_b x^2 - (\log_b 4 + \log_b y + \log_b z)$

$= \log_b 3 + \log_b x^2 - \log_b 4 - \log_b y - \log_b z$

$= \log_b 3 + 2\log_b x - \log_b 4 - \log_b y - \log_b z$

(d) $\log_b(x^2 y^4 z^7)$

$= 2\log_b x + 4\log_b y + 7\log_b z$

Ex: Write as a single logarithm

(a) $2\log_{12} x - 7\log_{12} y + \frac{1}{5}\log_3 w$

$= \log_{12} x^2 - \log_{12} y^7 + \log_3 w^{\frac{1}{5}}$

$= \log_{12}\left(\frac{x^2}{y^7}\right) + \log_3 w^{\frac{1}{5}}$

$\log_{12} x^2 + \log_{12} y^{-7} + \log_3 w^{\frac{1}{5}}$

$\log_{12}(x^2 y^{-7}) + \log_3 w^{\frac{1}{5}}$

(b) $\frac{1}{2}\log(x-2) - 3\log y + 17\log z$

$= \log\left(\frac{(x-2)^{1/2} z^{17}}{y^3}\right)$

NOTE: If the coefficient of a logarithm in expanded form is $+$, then the expression in the operand will be in the numerator of the condensed logarithm.
If the coefficient of a logarithm in expanded form is $-$, then the expression in the operand will be in the denominator of the condensed logarithm.

Ex: Suppose $\log_b 4 = 5$, $\log_b 9 = 10$ and $\log_b 7 = 8$. Find

(a) $\log_b 28 = \log_b(7\cdot 4) = \log_b 7 + \log_b 4$

$= 8 + 5$

$= 13$

(b) $\log_b\left(\frac{63}{4}\right) = \log_b\left(\frac{7\cdot 9}{4}\right)$

$= \log_b 7 + \log_b 9 - \log_b 4$

$= 8 + 10 - 5$

$= 13$

In soln, we consider $[H^+]$ & $[OH^-]$

- pH$= -\log[H^+]$ $= \log[H^+]^{-1} = \log\frac{1}{[H^+]}$

14 } basic
7 neutral
} acidic
0

Ex: What is the $[H^+]$ concentration for sour pickles with a pH of 3.2?

$pH = -\log[H^+]$

$3.2 = -\log[H^+]$

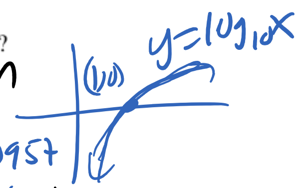
$-3.2 = \log_{10}[H^+]$

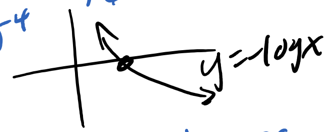
$10^{-3.2} = [H^+]$

$\log_b m = n \Leftrightarrow b^n = m$

So $[H^+] = .000630957$

$= 6.30957\times 10^{-4}$





$-3.2 = \log_{10}[H^+]$

$10^{-3.2} = [H^+]$

- Change of base formula: Suppose a, x, b are positive and $a \neq 1, b \neq 1$ Then, $\log_b x = \dfrac{\log_a x}{\log_a b}$

Ex: Find (a) $\log_3 52$

$\frac{\ln 52}{\ln 3} \approx 3.5966$

$= \frac{\log 52}{\log 3} \approx 3.5966$

$3^0 = 1$
$3^1 = 3$
$3^2 = 9$
$3^3 = 27$
$3^4 = 81$

(b) $\log_4 16$

$\log_4 16 = m \Leftrightarrow 4^m = 16$
$m = 2$

$\log_4 16 = \frac{\log 16}{\log 4} = 2$

$= \frac{\ln 16}{\ln 4} = 2$