Math 1314 – College Algebra
Section 6.5 Logarithmic Properties

- Recall: $\log_b m = n \iff$

Laws of Exponents: Let a and b be positive numbers and let x and y be real numbers. Then,

- $b^x b^y = b^{x+y}$
- $\dfrac{b^x}{b^y} = b^{x-y}$
- $(b^x)^y = b^{xy}$
- $(ab)^x = a^x b^x$
- $\left(\dfrac{a}{b}\right)^x = \dfrac{a^x}{b^x}$

Properties of Logarithms: If b, M, N are all positive numbers and $b \neq 1$, then

- $\log_b 1 = 0$
- $\log_b b = 1$
- $\log_b b^x = x$
- $b^{\log_b x} = x$
- $\log_b (MN) = \log_b M + \log_b N$
- $\log_b \left(\dfrac{M}{N}\right) = \log_b M - \log_b N$
- $\log_b M^p = p \log_b M$
- If $\log_b x = \log_b y$, then $x = y$.

- Recall: $y = b^x$ and $y = \log_b x$ are inverses of each other.
  Also, $(f \circ f^{-1})(x) = x$ and $(f^{-1} \circ f)(x) = x$ (property of inverses).

$x = (f \circ f^{-1})(x) = f(f^{-1}(x)) = f(\log_b x) = b^{\log_b x}$
$x = (f^{-1} \circ f)(x) = f^{-1}(f(x)) = f^{-1}(b^x) = \log_b b^x$

Ex: Simplify
(a) $\log_3 1$ (b) $\log_2 2$ (c) $7^{\log_7 13}$ (d) $\log_4 4^5$

Ex: Assume x, y, z, b are positive numbers and $b \neq 1$. Expand
(a) $\log_b (xyz)$ (b) $\log_b \left(\dfrac{xy}{z}\right)$

NOTE: If an expression is in the numerator of the coefficient of a logarithm in condensed form, the coefficient of the logarithm of that expression will be _____ when the logarithm is expanded.
If an expression is in the denominator of the coefficient of a logarithm in condensed form, the coefficient of the logarithm of that expression will be _____ when the logarithm is expanded.

(c) $\log_b \left(\dfrac{3x^2}{4yz}\right)$ (d) $\log_b (x^2 y^4 z^7)$

Ex: Write as a single logarithm

(a) $2\log_{12} x - 7\log_{12} y + \frac{1}{5}\log_3 w$ (b) $\frac{1}{2}\log(x-2) - 3\log y + 17\log z$

NOTE: If the coefficient of a logarithm in expanded form is _____, then the expression in the operand will be in the numerator of the condensed logarithm.
If the coefficient of a logarithm in expanded form is _____, then the expression in the operand will be in the denominator of the condensed logarithm.

Ex: Suppose $\log_b 4 = 5, \log_b 9 = 10$ and $\log_b 7 = 8$. Find

(a) $\log_b 28$ (b) $\log_b\left(\frac{63}{4}\right)$

- pH=$-\log[H^+]$

  Ex: What is the $[H^+]$ concentration for sour pickles with a pH of 3.2?

- Change of base formula: Suppose a, x, b are positive and $a \neq 1, b \neq 1$ Then, $\log_b x =$

  Ex: Find (a) $\log_3 52$ (b) $\log_4 16$