# Math 1314 – College Algebra
## Section 6.6 Exponential and Logarithmic Equations

Ex: Solve

No domain restrictions

(a) $2^{x^2+4x} = \frac{1}{8}$

$2^{x^2+4x} = 2^{-3}$

$x^2+4x = -3$

$x^2+4x+3=0$

$(x+1)(x+3)=0$

$x=-1$ or $x=-3$

1 3

No domain restrictions

(b) $2^{x^3-x} = 1$

$2^{x^3-x} = 2^0$

$x^3-x=0$

$x(x^2-1)=0$

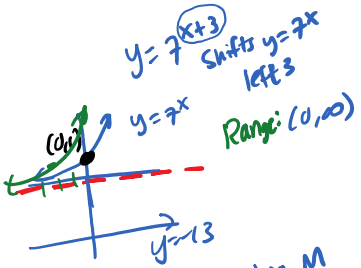
$x(x-1)(x+1)=0$

$x=0$ or $x=1, x=-1$

No domain restrictions

(c) $7^{x+3} = -13$

NO SOLN




$\log_b M^p = p\log_b M$

$3^0 = 1$

$3^1 = 3$

$2^2 = 9$

No domain restrictions

(d) $3^x = 7$

Take log of both sides

$\log(3^x) = \log(7)$

$\frac{x(\log 3)}{\log 3} = \frac{\log 7}{\log 3}$

$x = \frac{\log 7}{\log 3} \approx 1.7712$

Compare to:

$\frac{x(4)}{4} = \frac{25}{4}$

Ex: Solve

(a) $4^{x-2} = 7^{x+3}$

No domain restrictions

Take log of both sides:

$\log(4^{(x-2)}) = \log(7^{(x+3)})$

$(x-2)\log 4 = (x+3)\log 7$

$(\log 4)x - 2\log 4 = (\log 7)x + 3\log 7$

$(\log 4)x - (\log 7)x = 3\log 7 + 2\log 4$

$\frac{(\log 4 - \log 7)x}{\log 4 - \log 7} = \frac{3\log 7 + 2\log 4}{\log 4 - \log 7}$

$x = \frac{3\log 7 + 2\log 4}{\log 4 - \log 7} = \frac{\log 7^3 + \log 4^2}{\log(\frac{4}{7})} = \frac{\log(7^3 4^2)}{\log(\frac{4}{7})} = \frac{+\#}{-\#}$

$\approx -15.3861$

$\log_b M - \log_b N = \log_b(\frac{M}{N})$

Compare to:

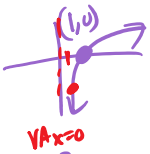
$(x-2)7 = (x+3)5$

$7x - 14 = 5x + 15$

$-5x + 14 \quad -5x + 14$

$\frac{2x}{2} = \frac{29}{2}$

$x = 14.5$




(b) $\frac{2e^{2x}}{2} = \frac{8e^{-6x}}{2}$

$e^{2x} = 4e^{-6x}$

Take ln of both sides:

$\ln(e^{2x}) = \ln(4e^{-6x})$

$2x(\ln e) = \ln 4 + \ln e^{(-6x)}$

$2x(\ln e) = \ln 4 - 6x(\ln e)$

$2x = \ln 4 - 6x$

$+6x \quad +6x$

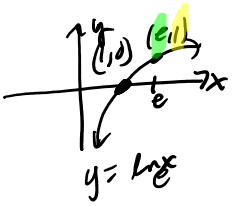
$8x = \ln 4$

$x = \frac{\ln 4}{8} \approx .1733$

Recall: $\ln x$ & $e^x$ are inverses of each other.

$e^{\ln \star} = \star \qquad \ln e^{\star} = \star$

$\log_b(MN) = \log_b M + \log_b N$




$\ln(e^{2x}) = \ln(4e^{-6x})$

$\ln(e^{2x}) = \ln 4 + \ln(e^{-6x})$

$2x = \ln 4 - 6x$

$\ln e^{\star} = \star$

Ex: Solve

(a) $\log_b(3x+2) - \log_b(2x-3) = 0$

$\log_b^m = n \Leftrightarrow b^n = m$

$\log_b\left(\frac{3x+2}{2x-3}\right) = 0$

$b^0 = \frac{3x+2}{2x-3}$

$(2x-3)\,1 = \frac{3x+2}{(2x-3)}(2x-3)$

$2x - 3 = 3x + 2$

$-3 = x + 2$

$-5 = x$

Domain restrictions: operand > 0

$3x+2 > 0 \qquad 2x-3 > 0$

$3x > -2 \qquad 2x > 3$

$x > -\frac{2}{3}$ AND $x > \frac{3}{2}$

Check potential soln against the restrictions:

Is $-5 > -\frac{2}{3}$? No.

Is $-5 > \frac{3}{2}$? No.

So $x = -5$ is an extraneous soln.

NO SOLN

(b) $\log_{\frac{1}{2}}(4x-5) = -3$

$\left(\frac{1}{2}\right)^{-3} = 4x - 5$

$2^3 = 4x - 5$

$8 = 4x - 5$

$13 = 4x$

$\frac{13}{4} = x$

Domain restrictions: operand > 0

$4x - 5 > 0$

$4x > 5$

$x > \frac{5}{4}$

Check potential soln w/ the domain restrictions:

Is $\frac{13}{4} > \frac{5}{4}$? Yes!

$x = \frac{13}{4}$

$y = e^x$ Domain: $\mathbb{R}$

$y = e^{-x}$

No domain issues

Ex: Solve

(a) $\log_7 x + \log_7 (x-5) = \log_7 6$

$\log_b M + \log_b N = \log_b(MN)$ $\quad \log_b(MN) = \log_b M + \log_b N$

Domain: operand $>0$

$x>0,\ x-5>0,\ 6>0$

$x>5$

$\log_7[x(x-5)] = \log_7 6$

$\log_7(x^2-5x) = \log_7 6$

$x^2-5x = 6$

$x^2-5x-6=0$ $\quad +1\ -6,\ +2\ -3$

$(x+1)(x-6)=0$

$x=-1$ or $x=6$ (extraneous soln → $x=-1$)

check w/ restrictions.

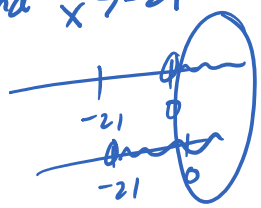
$-1 \not> 5$

(b) $\log x + \log(x+21) = 2$

$\log_b M + \log_b N = \log_b(MN)$

Domain: operand $>0$

$x>0$ and $x+21>0$, $x>-21$




$\log_{10}(x(x+21)) = 2$

$10^2 = x(x+21)$

$100 = x^2+21x$

$0 = x^2+21x-100$

$-1\ +100$
$-2\ +50$
$-4\ +25$
$-5\ +20$
$-10\ +10$

$0=(x-4)(x+25)$

$x=4$ or $x=-25$ (extraneous soln.)

check w/ restrictions

Ex: How old is a wooden statue that contains 72.1% of its original Carbon-14 content?

$72.1\% \Rightarrow .721$ $\quad 100\% \Rightarrow 1$

half-life of Carbon-14 is 5730

How old is box if 30% of C-14 content has decayed?

$A=.7$

$A_0=1$

$A = A_0(2^{-t/h})$

$A=.721$

$A_0=1$

$t=?$

$h=5730$

$.721 = 1(2^{-t/5730})$

$.721 = 2^{-t/5730}$ Solve for t

Take log of both sides:

$\log .721 = \log 2^{(-t/5730)}$

$\log .721 = \frac{-t}{5730}(\log 2)$ $\quad \cdot 5730$

$5730(\log .721) = -t(\log 2)$

$\frac{5730(\log .721)}{-\log 2} = \frac{-t(\log 2)}{-\log 2}$

$t = -\frac{5730\log .721}{\log 2}$

$t \approx 2704$ yrs.

$\frac{-5730(-)}{+}$

Round to nearest year

$\frac{-x(6)}{-6} = \frac{-6x}{-6}$