MATHEMATICAL MODELS

Ex: A small company manufactures puzzle games. Production per hour increases for a few hours before lunch, then decreases as the day goes on. The total number of puzzle games produced over a time period t in hours beginning at 7am, is $P(t) = -t^3 + 7t^2 + 8t, 0 \le t \le 5$.

(a) How many puzzle games are produced from 7am to 8am?

Video quiz

(b) How many puzzle games are produced from 7am to 9am?

(c) How many puzzle games are produced from 8am to 9am?

Ex: An apple orchard has an average yield of 36 bushels of apples/tree if tree density is 22 trees/acre. For each unit increase in tree density, the yield decreases by 2 bushels/tree. Letting x denote the number of trees beyond 22/acre, find a function in x that gives the yield of apples.

x = # of trees beyond 22 per acre

$f(x) = y$ = yield of apples in # of bushels (per acre)

average yield: $\left(\frac{36 \text{ bushels}}{\text{tree}}\right)\left(\frac{22 \text{ trees}}{1 \text{ acre}}\right) = \frac{\text{\# bushels}}{\text{acre}}$

add 1 tree per acre: $(36 - 2)(22 + 1)$

look for a pattern

add 2 trees/acre: $(36-2-2)(22+1+1)$

add 3 trees/acre: $(36-2-2-2)(22+1+1+1)$

$(36-2(3))(22+3)$

add 4 trees/acre: $(36-2(4))(22+4)$

5 trees/acre: $(36-2(5))(22+5)$

$\vdots$

x trees/acre: $(36-2x)(22+x)$

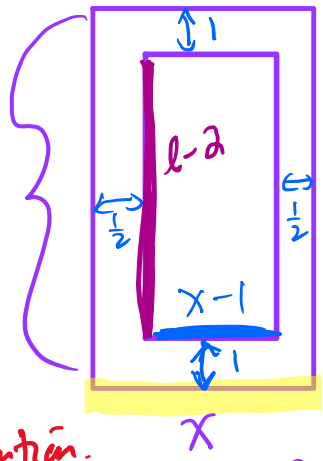
$$f(x) = (36-2x)(22+x)$$

yield of apples (units are # of bushels/acre)

Ex: A book designer has decided that the pages of a book should have 1 inch margins at the top and bottom of each page and $\frac{1}{2}$ inch margins on the sides. She also stipulated that each page should have a total area of 50 square inches. Find a function in the variable x, where x represents the width of a page, giving the area of the printed part of the page.

x = width of page in inches

$f(x) = y$ = area of printed part of page (in sq. inches)



$A_{page} = 50 \text{ in}^2 = \ell w$

$A_{page} = 50 \text{ in}^2 = x\ell$

Find a fcn in x ONLY. Get rid of ℓ.

Need to somehow write ℓ = x stuff from another piece of information & make a substitution.

$A_{printed} = (\ell - 2)(x-1)$ this ℓ is the problem!

$f(x) = A_{printed\ part} = \left(\frac{50}{x} - 2\right)(x-1)$

$\frac{50}{x} = \frac{x\ell}{x}$

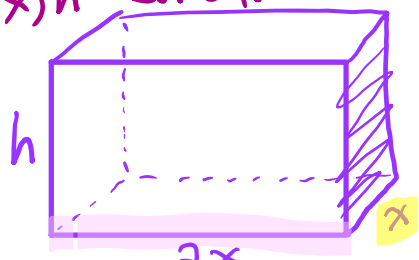
$\frac{50}{x} = \ell$

Ex: A rectangular storage container with an open top has a volume of 30 m^3. The length of its base is twice its width. Material for the base costs \$12 per square meter. Material for the sides costs \$8 per square meter. Express the cost of materials as a function of the width of the base, x.

$V = 30 \text{ m}^3 = \ell w h$

$30 = (2x)(x)h$ Solve for h

x = width of base in meters



Find cost.

1st: Think about surface area.

Front Back side side

SA: Base $(2x)(x)$ + $h(2x)$ + $h(2x)$ + xh + xh

m·m m·m 2xh 2xh xh xh

$\left(\frac{\$}{m^2}\right)(m^2) = \$$

Cost: Base $(2x^2)(12)$ + Sides $(6xh)(8)$

$(m^2)\left(\frac{\$}{m^2}\right)$ m^2 $\left(\frac{\$}{m^2}\right)$

Note: here, sides means front, back, & 2 sides

$C(x) = 24x^2 + 48xh$ Uhoh! We need a cost fcn w/ only x!!!

Get rid of h. h = x stuff

V: $30 = 2x^2 h$

$\frac{30}{2x^2} = \frac{2x^2 h}{2x^2}$

$\frac{15}{x^2} = h$

$C(x) = 24x^2 + 48x\left(\frac{15}{x^2}\right) \Rightarrow \boxed{C(x) = 24x^2 + \frac{720}{x},\ x > 0}$