

# MATH 1324 – FINITE MATHEMATICS PRE-CHAPTER 4 SETS AND SURVEY PROBLEMS

■ **Definition:** A set is a well-defined collection of objects. Denoted by capital letters. This is not just any collection. It must be defined in such a way that we can determine whether or not an object belongs to the collection.

■ Objects of a set are called elements. Usually denoted by lowercase letters.

Ex: Suppose  $A$  is the set containing the elements  $a, b,$  and  $c$ . We write  $A = \{a, b, c\}$

Ex: Suppose  $B$  is the set containing all the letters of the English alphabet. We write  $B = \{a, b, c, d, e, f, g, \dots, x, y, z\}$  OR  $B = \{x \mid x \text{ is lowercase letter of English alphabet}\}$

$B = \{a, b, c, d, e, f, g, \dots, x, y, z\}$  *roster notation* OR  $B = \{x \mid x \text{ is lowercase letter of English alphabet}\}$  *Set-builder notation*

■ If  $a$  is an element of a set  $A$ , we write  $a \in A$ . *such that or given* If  $a$  is not in  $A$ , we write  $a \notin A$

■ Two sets  $A$  and  $B$  are equal if and only if they contain the same elements. Written as  $A = B$ .

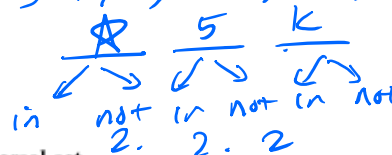
⇒ ■ If every element of a set  $A$  is also an element of a set  $B$ , then  $A$  is a subset of  $B$ . We write  $A \subseteq B$

■ NOTE: Every set  $A$  is a subset of itself.  $A \subseteq A$   $\emptyset \subseteq \emptyset \subseteq \emptyset \subseteq \emptyset$

■ The set that contains no elements is the empty set or null set. Denoted by  $\emptyset$ . The empty set is a subset of every set.

Ex: List all subsets of the set  $A = \{x, 5, k\}$   $\emptyset, \{x\}, \{5\}, \{k\}, \{x, 5\}, \{x, k\}, \{5, k\}, \{x, 5, k\}$

$2^{\# \text{ of elements}} = \# \text{ of subsets}$

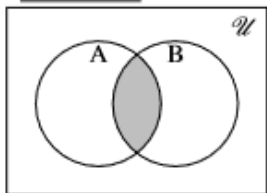


■ The set of all elements of interest in a particular situation is the universal set.

*proper subset: A ⊂ B if all elements in A are in B & there are more in B.*

■ Venn Diagrams

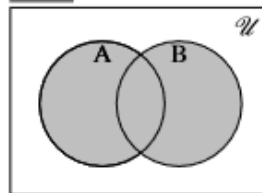
■ Intersection of sets  $A$  and  $B$ :



$\cup$   
and

$A \cap B = \{x \mid x \in A \text{ and } x \in B\}$   
The set of all elements in both  $A$  and  $B$ .

■ Union of sets  $A$  and  $B$ :



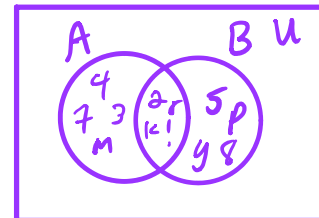
OR

$A \cup B = \{x \mid x \in A, x \in B, x \in A \cap B\}$   
The set of all elements that belong to either  $A$  or  $B$  or both.

Ex: Let  $A = \{2, 4, r, 7, m, !, k, 3\}, B = \{5, 2, p, r, k, !, 8, y\}$ . Find

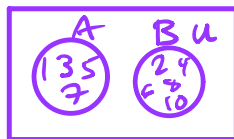
(a)  $A \cup B = \{2, 4, r, 7, m, !, k, 3, 5, p, 8, y\}$

(b)  $A \cap B = \{2, r, k, !\}$



Ex: Let  $A = \{1, 3, 5, 7\}$  and  $B = \{2, 4, 6, 8, 10\}$ . Find

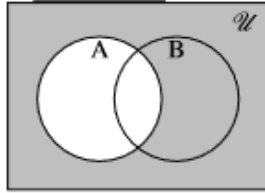
(a)  $A \cup B = \{1, 3, 5, 7, 2, 4, 6, 8, 10\}$



(b)  $A \cap B = \emptyset$  empty set

$A$  &  $B$  are Disjoint  $A \cap B = \emptyset$

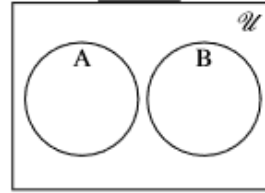
■ **Complement of set A:**



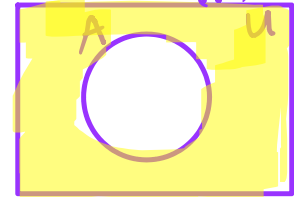
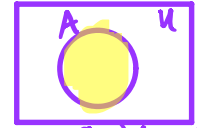
$$A^c = \{x \mid x \in U \text{ and } x \notin A\}$$

The set of all elements in  $U$  but not in  $A$ .

■ **Disjoint:**



$$A \cap B = \emptyset$$



$$A^c = \emptyset$$

- Sets  $A$  and  $B$  are disjoint if they have no elements in common. (If  $A \cap B = \emptyset$ ).

Ex: Suppose  $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$  and  $A = \{2, 3, 6, 8\}$ . Find  $A^c$ .

$$A^c = \{1, 4, 5, 7, 9, 10\}$$

- **Set Complementation:** If  $U$  is a universal set and  $A$  is a subset of  $U$ , then

■  $U^c = \emptyset$       ■  $\emptyset^c = U$       ■  $(A^c)^c = A$       ■  $A \cup A^c = U$       ■  $A \cap A^c = \emptyset$

- **Set Operations:** Let  $U$  be a universal set. If  $A, B, C$  are arbitrary subsets of  $U$ , then

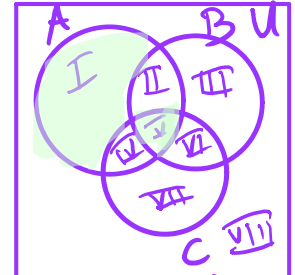
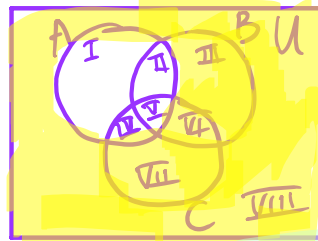
■  $A \cup B = B \cup A$       *Commutative*  $\Rightarrow$       ■  $A \cap B = B \cap A$   
 ■  $A \cup (B \cap C) = (A \cup B) \cap C$       *Associative*  $\rightarrow$       ■  $A \cap (B \cap C) = (A \cap B) \cap C$   
 ■  $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$       *distributive*  $\rightarrow$       ■  $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

$$3(x + 5) = 3x + 2(5)$$

- **De Morgan's Laws:** Let  $A$  and  $B$  be sets. Then  $(A \cup B)^c = A^c \cap B^c$  and  $(A \cap B)^c = A^c \cup B^c$

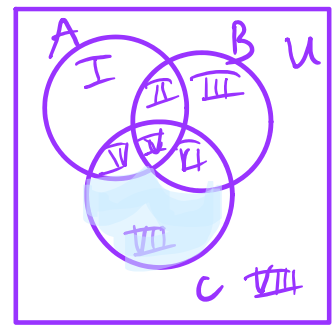
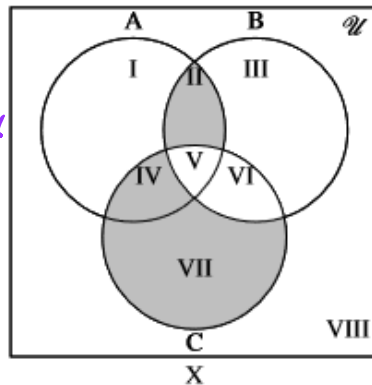
Ex: Given the set  $X$  below, find

- $X \cup A^c$
- $(X \cup A) \cup (C^c \cap B)$
- $(X \cup A)^c \cap B$
- $(X \cup A)^c \cup B$



$(X \cup A^c)^c$   
 Shade a region only if it is shaded in BOTH sets

$\cup$   
 Shade if in either set



- Problems that call for finding the number of elements in a set are called counting problems. This field of study is called combinatorics.
- The number of elements in a finite set is determined by simply counting the elements in the set.
- If  $A$  is a set,  $n(A)$  denotes the number of elements in  $A$ .

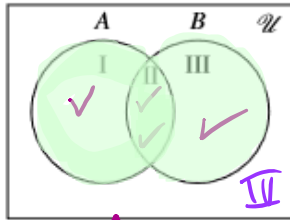
Recall: if A & B are disjoint:  $A \cap B = \emptyset$

■  $n(\emptyset) = 0$

■ If A and B are not disjoint sets, then

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

b/c we don't count elements in the intersection



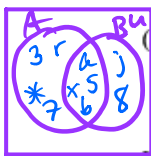
$A \cup B$

■ If A and B are disjoint sets, then

$A \cap B = \{a, 5, b\} \times 3$

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

Ex: Suppose  $A = \{3, r, a, 5, 7, *, b, x\}$ ,  $B = \{a, x, 5, b, j, 8\}$ ,  $C = \{r, a, 7\}$ , and  $D = \{3, b, *\}$ . Find



(a)  $n(A \cup B) = n(A) + n(B) - n(A \cap B)$   
 $= 8 + 6 - 4 = 10$

(b)  $n(C \cup D) = n(C) + n(D) - n(C \cap D)$   
 $= 3 + 3 - 0 = 6$

Ex: Among 500 freshmen pursuing a business degree at a university, 320 are enrolled in an economics course, 225 are enrolled in a mathematics course, and 140 are enrolled in both an economics and a math course.

(a) How many freshmen from this group are enrolled in an economics or a math course?

$180 + 140 + 85$

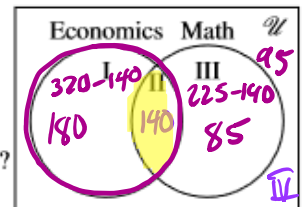
$n(\text{Econ} \cup \text{Math}) = n(\text{Econ}) + n(\text{Math}) - n(\text{Econ} \cap \text{Math})$   
 $= 320 + 225 - 140 = 405$

(b) How many freshmen from this group are enrolled in only an economics course?

Region I: 180

(c) How many freshmen from this group are enrolled in only an economics but not a math course?

Region I: 180



Ex: A survey of 100 teenagers about their hobbies revealed the following:

- ✓ 40 play video games
- ✓ 9 only play sports
- ✓ 25 read novels
- ✓ 15 play video games and play sports
- ✓ 12 play video games and read novels
- ✓ 10 play sports and read novels
- ✓ 4 read all three

Start w/ Region 5  
 How many like sports, video games or reading?

(a) How many teenagers surveyed have at least one of those hobbies?

Regions I, II, III, IV, V, VI, VII: 62

(b) How many teenagers surveyed have exactly one of those hobbies?

$17 + 9 + 7 = 33$

(c) How many teenagers surveyed have exactly two of those hobbies?

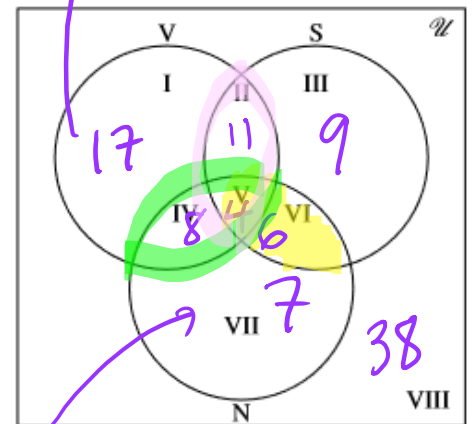
II, IV, VI:  $11 + 8 + 6 = 25$

(d) How many teenagers surveyed have none of those hobbies?

Region VIII: 38

(e) How many teenagers surveyed play video games but do not read novels?

Regions I & II:  $17 + 11 = 28$



$25 - 8 - 4 - 6$

Region VIII:  $100 - 17 - 11 - 9 - 8 - 4 - 6 - 7$



- Ex:  
 In a survey of 300 people:  
 ✓ 122 drink Sprite  
 ✓ 150 drink Coke  
 ✓ 62 drink Pepsi  
 ✓ 38 drink Sprite and Coke  
 ✓ 20 drink Sprite and Pepsi  
 ✓ 28 drink Coke and Pepsi  
 ✓ 36 drink none of these.  
 Fill in the Venn diagram.

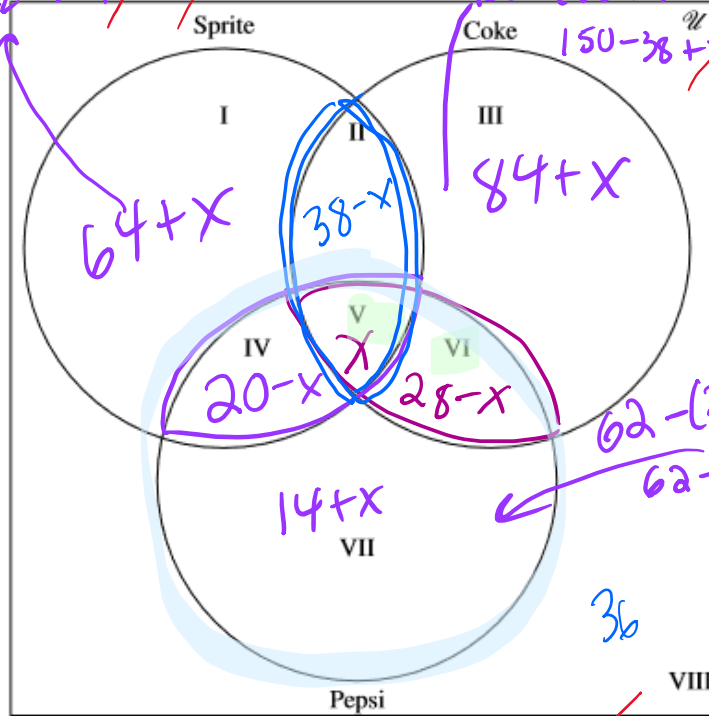
$$122 - (38-x) - x - (20-x)$$

$$122 - 38 + x - x - 20 + x$$

$$150 - (38-x) - x - (28-x)$$

$$150 - 38 + x - x - 28 + x$$

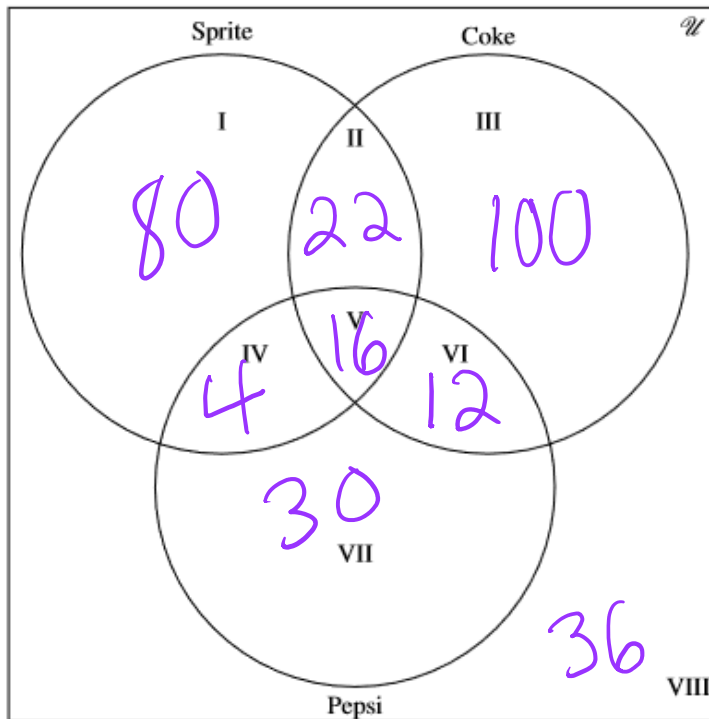
$$84 + x$$



$$62 - (20-x) - x - (28-x)$$

$$62 - 20 + x - x - 28 + x$$

$$14 + x$$



$$\begin{array}{r}
 64+x \\
 + 38-x \\
 + 84+x \\
 + 20-x \\
 + \quad \quad x \\
 + 28-x \\
 + 14+x \\
 + 36 \\
 \hline
 300
 \end{array}$$

$$284 + x = 300$$

$$x = 16$$