MATH 1324 – FINITE MATHEMATICS
SECTION 1.1 BASIC MATRIX OPERATIONS

- Matrix: ordered rectangular array of numbers. Matrices are denoted by capital letters
  A matrix A with m rows and n columns has size $m \times n$. The entry in the ith row and jth column is denoted by a_{ij}.

  # of rows # of columns

  $a_{1,1}$ or a_{1-1}

  $$A = \begin{bmatrix} 7 & 3 & 6 \\ 12 & -1 & 13 \end{bmatrix}$$

  Size: 2×3

  $a_{11} = 7$ row 1 col 1

  $a_{14} = X$ row 1 col 4

  $a_{13} = 6$

  $a_{23} = 13$ row 2 col 3

  $a_{22} = -1$

- Row matrix: matrix of size $1 \times n$

  $B = [\ \ x \ \ k \ \ \ell \ \ 13]$ Size: 1×5

- Column matrix: size $m \times 1$

  $$C = \begin{bmatrix} 12 \\ 3 \\ 8 \end{bmatrix}$$ Size: 3×1

- Two matrices are equal if their corresponding entries are equal
  ☆☆☆ MUST be the same size ☆☆☆

- Matrix Operations:

1. Addition: add corresponding entries
   NOTE: ⋆⋆⋆ Can only add two matrices if they are the same size ⋆⋆⋆

   $O_{2\times 2} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

   Properties for Matrix Addition: For all matrices of size $m \times n$:
   - Commutativity $A + B = B + A$
   - Associativity $A + (B + C) = (A + B) + C$
   - Identity $A + O_{m\times n} = A$
   - Additive inverse

   $A + (-A) = O_{m \times n}$

   $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ $B = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$

   $A + B = \begin{bmatrix} 1+5 & 2+6 \\ 3+7 & 4+8 \end{bmatrix} = \begin{bmatrix} 6 & 8 \\ 10 & 12 \end{bmatrix}$

2. Subtraction: Subtract corresponding entries
   NOTE: ⋆⋆⋆ Can only subtract two matrices if they are the same size ⋆⋆⋆

   $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$ $A^T = \begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix}$ Size: 2×3 or $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$ $A^T = \begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix}$

   Size: 3×2

3. Transpose: The transpose of an $m \times n$ matrix A with entries a_{ij} is the $n \times m$ matrix A^T with entries a_{ji}.

   $a_{21} = 3$ $a^T_{12} = 3$

4. Scalar product: Let c be a real number (scalar). cA is found by multiplying every entry in matrix A by c.

   $$3A = 3\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} = \begin{bmatrix} 3(1) & 3(2) \\ 3(3) & 3(4) \\ 3(5) & 3(6) \end{bmatrix} = \begin{bmatrix} 3 & 6 \\ 9 & 12 \\ 15 & 18 \end{bmatrix}$$

- NOTE: Remember we cannot use the calculator if one or more of the matrices contains variable entries.

Ex: Given $A = \begin{bmatrix} 5 & 3 & 1 \\ 4 & 0 & 2 \end{bmatrix}$ $B = \begin{bmatrix} 4 & -3 \\ -2 & 6 \\ 1 & 2 \end{bmatrix}$ $C = \begin{bmatrix} -8 \\ 2 \\ 7 \end{bmatrix}$ $D = \begin{bmatrix} 5 & 7 & 3 \end{bmatrix}$.

(a) Find $a_{12} = 3$

(b) Find $b_{32} = 2$

(c) Find c_{13} X

(d) Which is the column matrix? C   Row matrix: D

(e) Find B^T. $B^T = \begin{bmatrix} 4 & -2 & 1 \\ -3 & 6 & 2 \end{bmatrix}$

$A = \begin{bmatrix} 5 & 3 & 1 \\ 4 & 0 & 2 \end{bmatrix}$ $B = \begin{bmatrix} 4 & -3 \\ -2 & 6 \\ 1 & 2 \end{bmatrix}$

(f) Compute $A + 3B$

2×3 3×2 X   cannot add these. Must be the same size.

$A^T + 3B$

(g) Compute $A^T + 3B$.

3×2 3×2 ✓

$$\begin{bmatrix} 5 & 4 \\ 3 & 0 \\ 1 & 2 \end{bmatrix} + 3\begin{bmatrix} 4 & -3 \\ -2 & 6 \\ 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 4 \\ 3 & 0 \\ 1 & 2 \end{bmatrix} + \begin{bmatrix} 12 & -9 \\ -6 & 18 \\ 3 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 17 & -5 \\ -3 & 18 \\ 4 & 8 \end{bmatrix}$$

(h) If $D = \begin{bmatrix} z+3 & x+z & y \end{bmatrix}$ find x, y, z. $D = \begin{bmatrix} 5 & 7 & 3 \end{bmatrix}$

$$\begin{bmatrix} z+3 & x+z & y \end{bmatrix} = \begin{bmatrix} 5 & 7 & 3 \end{bmatrix}$$

11 entry: $z+3 = 5$

12 entry: $x+z = 7$

13 entry: $y = 3$

$\boxed{x=5 \quad y=3 \quad z=2}$

$\cancel{z+3} = 5$
$-3 \quad -3$

$z = 2$

$x + z = 7$

$x + \cancel{2} = 7$
$-2 \quad -2$

$x = 5$

$y = 3$

$$\boxed{z = 2}$$

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Ex: Given matrix equation $3A - 2B = C$ below, find a and b:

$$3\begin{bmatrix} 4 & a & 0 \\ 1 & 4 & 1 \end{bmatrix} - 2\begin{bmatrix} 6 & 3 & 9 \\ 2 & 1 & b \end{bmatrix} = \begin{bmatrix} 0 & 0 & -18 \\ -1 & 10 & 8 \end{bmatrix}$$

$$3\begin{bmatrix} 4 & a & 0 \\ 1 & 4 & 1 \end{bmatrix} + (-2)\begin{bmatrix} 6 & 3 & 9 \\ 2 & 1 & b \end{bmatrix} = \begin{bmatrix} 0 & 0 & -18 \\ -1 & 10 & 8 \end{bmatrix}$$

$$\begin{bmatrix} 12 & 3a & 0 \\ 3 & 12 & 3 \end{bmatrix} + \begin{bmatrix} -12 & -6 & -18 \\ -4 & -2 & -2b \end{bmatrix} = \begin{bmatrix} 0 & 0 & -18 \\ -1 & 10 & 8 \end{bmatrix}$$

$$\begin{bmatrix} 12-12 & 3a-6 & 0-18 \\ 3-4 & 12-2 & 3-2b \end{bmatrix} = \begin{bmatrix} 0 & 0 & -18 \\ -1 & 10 & 8 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 3a-6 & -18 \\ -1 & 10 & 3-2b \end{bmatrix} = \begin{bmatrix} 0 & 0 & -18 \\ -1 & 10 & 8 \end{bmatrix}$$

11 entry: $0 = 0$ ✓

12 entry: $3a - 6 = 0$
$+6 \quad +6$
$\frac{3a}{3} = \frac{6}{3}$
$\boxed{a = 2}$

13 entry: $-18 = -18$ ✓

21 entry: $-1 = -1$ ✓

22 entry: $10 = 10$ ✓

23 entry: $3 - 2b = 8$
$-3 \quad\quad -3$
$\frac{-2b}{-2} = \frac{5}{-2}$
$\boxed{b = -\frac{5}{2}}$

$3 + (-2b) = 8$
$-3 \quad\quad -3$