$A_{m\times n} B_{n\times p} = C_{m\times p}$

MATH 1324 – FINITE MATHEMATICS
SECTION 1.2 MATRIX MULTIPLICATION

- To multiply AB: Check to see if it is possible: The number of columns of matrix A = the number of rows of matrix B. (i.e. when written together, the "middle" numbers must match.)

Ex:
- only use ROWS in 1st matrix
- only use COLUMNS in 2nd matrix
- 1st(1st) + 2nd(2nd) + 3rd(3rd) + 4th(4th) + ...

$$\underset{\text{Size: } 2\times 3}{\begin{bmatrix} 2 & 6 & 12 \\ 8 & x & 0 \end{bmatrix}} \underset{\text{Size: } 3\times 4}{\begin{bmatrix} 4 & m & 16 & 5 \\ 7 & p & -1 & 0 \\ 9 & 12 & 20 & w \end{bmatrix}} = \underset{\text{Size: } 2\times 4}{\begin{bmatrix} 158 & - & - & 10+12w \\ - & - & 128-x & - \end{bmatrix}}$$

$C_{23} = 8(16) + x(-1) + 0(20) = 128 - x$

use Row 2 in A; use Col 3 in B

$C_{14} = 2(5) + 6(0) + 12(w) = 10 + 12w$

Row 1 in A; Col 4 in B

$C_{11} = 2(4) + 6(7) + 12(9) = 8 + 42 + 108 = 158$

Row 1 in A; Col 1 in B

$A_{2\times 3} B_{3\times 4}$ ✓

$B_{3\times 4} A_{2\times 3}$ ✗

NOTE: Commutative property does not hold.

- Laws for Matrix Multiplication: If products and sums are defined for matrices A, B, C then the following hold:
  1. Associativity $(AB)C = A(BC)$
  2. Distributive Property $A(B+C) = AB + AC$
- NOTE: Remember we cannot use the calculator if one or more of the matrices contains variable entries.
- Identity matrix:

$$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad I_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Square Matrix

If # rows = # of columns

$$I_n = \begin{bmatrix} 1 & & & \\ & 1 & & \\ & & \ddots & \\ & & & 1 \end{bmatrix}$$

$A = \begin{bmatrix} 6 & 9 & 3 \\ 13 & 27 & 8 \end{bmatrix}$ $A_{2\times 3} I_3$

Size: 2x3

$\begin{bmatrix} 6 & 9 & 3 \\ 13 & 27 & 8 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 6 & 9 & 3 \\ 13 & 27 & 8 \end{bmatrix}$

2x3 · 3x3

$13(1) + 27(0) + 8(0)$

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 6 & 9 & 3 \\ 13 & 27 & 8 \end{bmatrix} = \begin{bmatrix} 6 & 9 & 3 \\ 13 & 27 & 8 \end{bmatrix}$

2x2 · 2x3 → $I_2 A_{2\times 3}$

Ex: Given $A = \begin{bmatrix} 5 & 3 \\ 4 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & 6 \\ 1 & 2 \end{bmatrix}$,

(a) find AB

$A_{2\times 2} B_{2\times 2} = C_{2\times 2}$

$\begin{bmatrix} 5 & 3 \\ 4 & 0 \end{bmatrix}\begin{bmatrix} -2 & 6 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} -7 & 36 \\ -8 & 24 \end{bmatrix}$ (A, B, C)

$c_{11} = 5(-2)+3(1) = -10+3 = -7$ $\quad c_{12} = 5(6)+3(2) = 30+6 = 36$

$c_{21} = 4(-2)+0(1) = -8$ $\quad c_{22} = 4(6)+0(2) = 24$

(b) find BA

$\begin{bmatrix} -2 & 6 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 5 & 3 \\ 4 & 0 \end{bmatrix} = \begin{bmatrix} 14 & -6 \\ 13 & 3 \end{bmatrix}$ (B, A, C)

$2\times 2 \quad 2\times 2$

$c_{11} = (-2)(5)+6(4) = -10+24 = 14$ $\quad c_{12} = (-2)(3)+6(0) = -6$

$c_{21} = 1(5)+2(4) = 5+8 = 13$ $\quad c_{22} = 1(3)+2(0) = 3$

$AB \neq BA$

(c) find $(A+B)^2$

$\begin{bmatrix} 5 & 3 \\ 4 & 0 \end{bmatrix} + \begin{bmatrix} -2 & 6 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 3 & 9 \\ 5 & 2 \end{bmatrix}$ (A, B, A+B)

$(A+B)(A+B)$

$\begin{bmatrix} 3 & 9 \\ 5 & 2 \end{bmatrix}\begin{bmatrix} 3 & 9 \\ 5 & 2 \end{bmatrix} = \begin{bmatrix} 54 & 45 \\ 25 & 49 \end{bmatrix}$ (C)

$2\times 2 \quad 2\times 2$

$c_{11} = 3(3)+9(5) = 9+45$ $\quad c_{12} = 3(9)+9(2) = 27+18 = 45$

$c_{21} = 5(3)+2(5) = 15+10 = 25$ $\quad c_{22} = 5(9)+2(2) = 45+4 = 49$

NOTE: To square a matrix write it twice & multiply if possible

$A = \begin{bmatrix} 6 & 9 & 3 \\ 13 & 22 & 8 \end{bmatrix}$ cannot be squared. $(2\times 3)(2\times 3)$