$\sum \frac{N}{0} \times \quad \frac{0}{K} = 0$

SECTION 2.1 REVIEW OF LINES

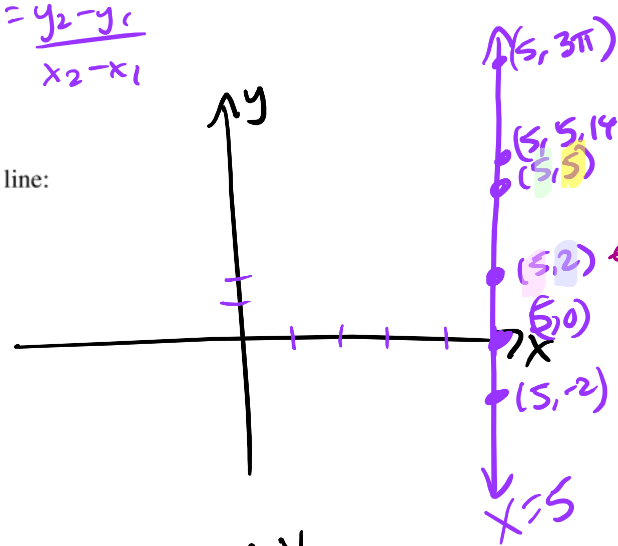
- Many important models are linear, which means the graph of the model is a straight line.
- SLOPE: $m = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x} = \frac{\text{change in } y}{\text{change in } x}$ = rate of change of y w/ respect to x $= \frac{y_2 - y_1}{x_2 - x_1}$

  The slope of all non-vertical lines is

  Point 1: (x_1, y_1) Point 2: (x_2, y_2)

  $m = \frac{y_2 - y_1}{x_2 - x_1}$

- Vertical line:



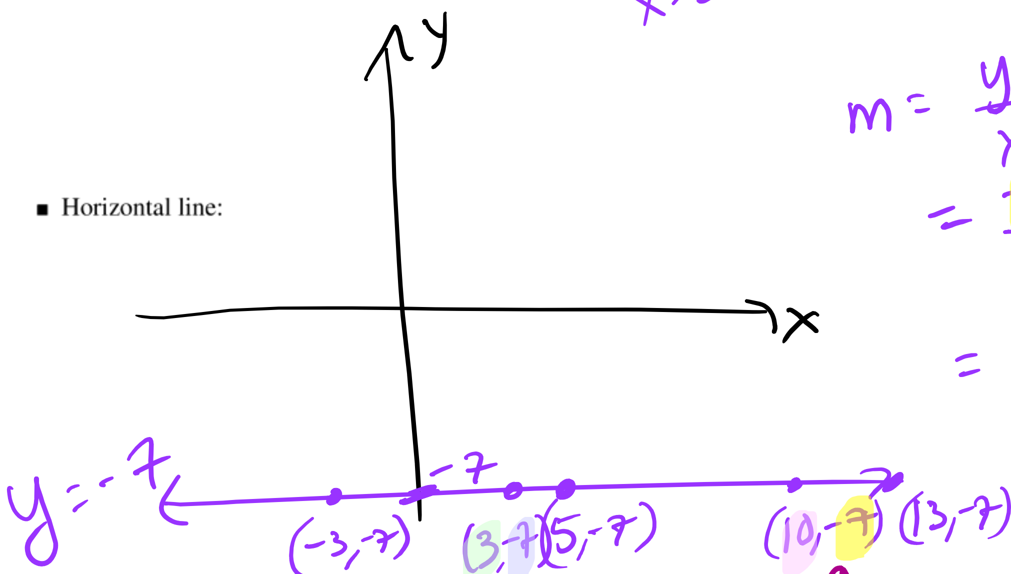
$m = \frac{y_2 - y_1}{x_2 - x_1}$ vinculum

$= \frac{5 - 2}{5 - 5} = \frac{3}{0}$

Slope is undefined.

☆ All vertical lines have form $x = a$ and have undefined slope.

- Horizontal line:



$m = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{-7 - (-7)}{10 - 3}$

$= \frac{-7 + 7}{10 - 3} = \frac{0}{7}$

$= 0$

☆ All horizontal lines have form $y = b$ and have slope of 0.

y-coordinate of y-int $(0, b)$

NOTE: It does not matter which order you put the points in the formula, as long as you stay consistent.

- LINES:
  - $y = mx + b$ Slope-intercept form
  - $y - y_1 = m(x - x_1)$ Point-slope form
  - $Ax + By = C$ Standard form
  - $Ax + By + D = 0$ General form

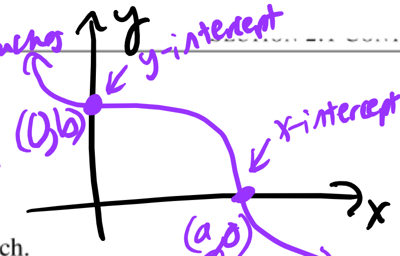
  no fractions and $A > 0$

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$m(x - x_1) = \frac{(y - y_1)}{\cancel{(x - x_1)}} \cancel{(x - x_1)}$

$m(x - x_1) = y - y_1$

$y - y_1 = m(x - x_1)$



- x-intercept: The point $(a,0)$ where the curve crosses/touches the x-axis ($y=0$ there)

y-intercept: The point $(0,b)$ where the curve crosses/touches the y-axis ($x=0$ there)

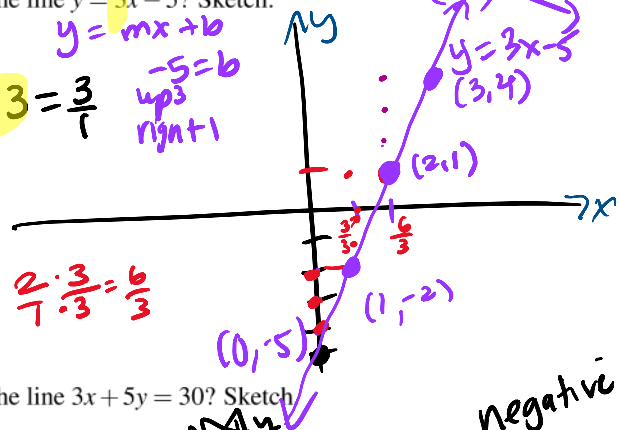
Ex: What are the x-intercept and y-intercept of the line $y = 3x - 5$? Sketch.

- x-int: Set $y=0$:
  $0 = 3x-5$ Solve for x.
  $+5 \quad +5$
  $\frac{5}{3} = \frac{3x}{3}$
  $x = \frac{5}{3}$
  $(\frac{5}{3}, 0)$

$y = mx + b$
$-5 = b$
$m = 3 = \frac{3}{1}$ up 3 right 1

$\frac{2}{1} \cdot \frac{3}{3} = \frac{6}{3}$

- y-int: Set $x=0$: $(0,-5)$
  $y = 3(0) - 5 \Rightarrow y = -5$



Ex: What are the x-intercept and y-intercept of the line $3x+5y = 30$? Sketch.

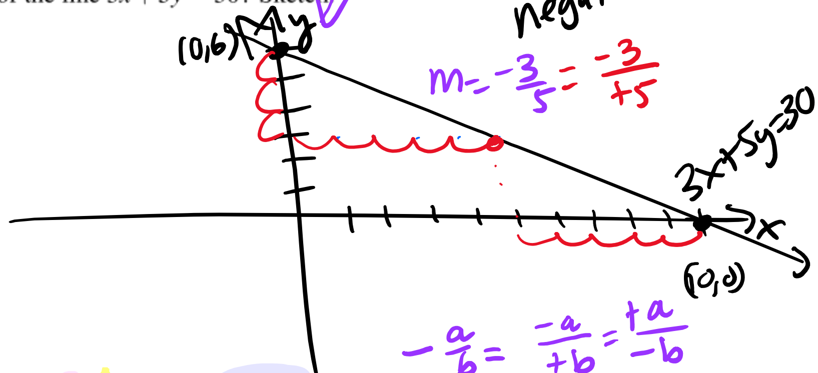
- x-int: Set $y=0$
  $3x+5(0)=30$ $(10,0)$
  $\frac{3x}{3} = \frac{30}{3}$
  $x=10$

- y-int: set $x=0$ $(0,6)$
  $3(0)+5y=30$
  $\frac{5y}{5} = \frac{30}{5}$ $y=6$

negative slope

$m = -\frac{3}{5} = \frac{-3}{+5}$

$-\frac{a}{b} = \frac{-a}{+b} = \frac{+a}{-b}$

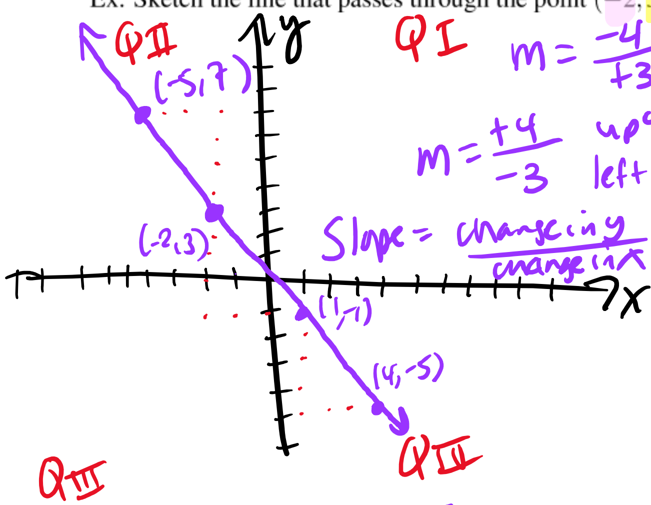


Ex: Sketch the line that passes through the point $(-2, 3)$ and has slope $-\frac{4}{3}$. Find the equation of the line.

$m = \frac{-4}{+3}$ down 4 right 3

$m = \frac{+4}{-3}$ up 4 left 3

Slope = $\frac{\text{change in y}}{\text{change in x}}$



What is the y-int?

$(0, \frac{1}{3})$

- Given a point & slope $\Rightarrow$ use point-slope form.

$y - y_1 = m(x - x_1)$

$y - 3 = -\frac{4}{3}(x - (-2))$ $\quad -\frac{4}{3} \cdot \frac{2}{1}$

$y - 3 = -\frac{4}{3}(x+2)$

$y - 3 = -\frac{4}{3}x - \frac{8}{3}$
$+3 \qquad +3$

$y = -\frac{4}{3}x - \frac{8}{3} + \frac{3}{1} \cdot \frac{3}{3}$

$y = -\frac{4}{3}x - \frac{8}{3} + \frac{9}{3}$

$\boxed{y = -\frac{4}{3}x + \frac{1}{3}}$

Ex: Find the equation of the line that passes through the points $(4,2)$ and $(6,10)$.

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{10-2}{6-4} = \frac{8}{2} = 4$

Point-slope: $y - y_1 = m(x - x_1)$

$y - 2 = 4(x - 4)$

$y - 2 = 4x - 16$

$+2 \qquad +2$

$\boxed{y = 4x - 14}$

What is the y-int: $(0, -14)$

- Parallel lines: never cross. have the same slope.

  $L_1 \parallel L_2$ if $m_1 = m_2$

- Perpendicular lines: cross at 90° angles. Slopes are negative reciprocals

  $L_1 \perp L_2$ if $m_1 = -\frac{1}{m_2}$ or $m_1 m_2 = -1$

or Ex: $m = -\frac{3}{2}$, $m_\perp = +\frac{2}{3}$

Ex: $m_1 = 4$, $m_\perp = -\frac{1}{4}$

Ex: Find the equation of the line that passes through the point $(0,8)$ that is parallel to the line $3x + 5y = 30$.

Need slope: Consider $\parallel$ line:

$3x + 5y = 30$. change to slope-int form.

$-3x \qquad -3x$

$\parallel$ line:

$\frac{5y}{5} = \frac{-3x}{5} + \frac{30}{5}$

$y = -\frac{3}{5}x + 6$

$m_\parallel = -\frac{3}{5}$

Our line: $m = -\frac{3}{5}$ & we have a point $(0,8)$

$y - y_1 = m(x - x_1)$

$y - 8 = -\frac{3}{5}(x - 0)$

$y - 8 = -\frac{3}{5}x$

$+8 \qquad +8$

$\boxed{y = -\frac{3}{5}x + 8}$

What is the y-int? $(0,8)$

$(0,8)$ & $m = -\frac{3}{5}$

$y = mx + b$

$y = -\frac{3}{5}x + 8$

Ex: Find the equation of the line passing through $(0,-8)$ that is perpendicular to the line passing through $(4,2)$ and $(6,10)$.

Our line: point $(0,-8)$

NEED SLOPE!

$\perp$ line: $(4,2)$ & $(6,10)$ or $\frac{2-10}{4-6} = 4$

$m_\perp = \frac{10-2}{6-4} = \frac{8}{2} = 4$

$m_\perp = 4$

Our line: $m = -\frac{1}{4}$

Our line: $m = -\frac{1}{4}$ & $(0,-8)$

$\boxed{y = -\frac{1}{4}x - 8}$

OR $y - y_1 = m(x - x_1)$

$y + 8 = -\frac{1}{4}(x - 0)$

$y + 8 = -\frac{1}{4}x$

$y = -\frac{1}{4}x - 8$

Ex: Given the line $6x + 8y = 16$, if x increases by 6 units, what is the corresponding change in y?

$\boxed{y \text{ decreases by } \tfrac{9}{2} \text{ units}}$

Recall: $m = \frac{\Delta y}{\Delta x} = \frac{\text{change in } y}{\text{change in } x}$

$m = \frac{\Delta y}{+6}$

Find m: $6x + 8y = 16 \Rightarrow$ change to slope-intercept $y = mx + b$

$-6x \qquad -6x$

$\frac{8y}{8} = \frac{-6x}{8} + \frac{16}{8}$

$y = -\frac{3}{4}x + 2 \Rightarrow$ coeff of x is m $m = -\frac{3}{4}$

$m = \frac{\Delta y}{+6}$

$-\frac{3}{4} = \frac{\Delta y}{6}$

$-\frac{3}{4}(6) = \frac{\Delta y}{\cancel{6}}\frac{(\cancel{6})}{1}$

$-\frac{18}{4} = \Delta y$

$-\frac{9}{2} = \Delta y$