# Section 2.2 Modeling with Linear Functions

f of x

- A function $f$ is a rule that assigns to each value of $x$ one and only one value of $y$. Write $y = f(x)$

output input

y depends on x

- The independent variable is __$x$__ and the dependent variable is __$y$__.
- The set of input values (the set of all possible $x$ values) is the __domain__.
- The set of output values (the set of all possible $y$ values) is the __range__.
- NOTE: $f(x) = y$ can be thought of as a point $(x, y)$. Ex: $f(1) = 5$ corresponds to the point $(1, 5)$.
- Function notation: We write $y = f(x)$ to show that $y$ is a function of $x$.
- The function $f$ defined by $f(x) = mx + b$, where $m$ and $b$ are constants is called a linear function.
- There are many types of functions.

Ex: Given $f(x) = 6x - 2$, find

Find y-value of $y=6x-2$ if $x=0$

(a) $f(0) = 6(0) - 2$
$f(0) = -2$

Point $(0,-2)$ is on the line $y = 6x - 2$
$f(x) = 6x - 2$

(b) $f(-6) = 6(-6) - 2$
$f(-6) = -38$

pt $(-6,-38)$ is on the line

(c) $f(100) = 6(100) - 2$
$f(100) = 598$

(d) $f(\star) = 6\star - 2$

Ex: Given the function $g(t) = \dfrac{3-5t}{(3t+2)^2}$, do the points $\left(-2, \dfrac{13}{16}\right)$ and $\left(\dfrac{3}{5}, \dfrac{1}{9}\right)$ lie on the graph of $g$?

input is $t$

$g(-2) = \dfrac{3-5(-2)}{(3(-2)+2)^2} = \dfrac{3+10}{(-6+2)^2} = \dfrac{13}{(-4)^2} = \dfrac{13}{16}$ ✓

yes! $(-2, \frac{13}{16})$ is on graph of g.

$g\left(\frac{3}{5}\right) = \dfrac{3 - \frac{5}{1}\left(\frac{3}{5}\right)}{\left(\frac{3}{1}\left(\frac{3}{5}\right)+2\right)^2} = \dfrac{3-3}{\left(\frac{9}{5}+2\right)^2} = \dfrac{0}{\#} = 0$

NOPE. $(\frac{3}{5}, \frac{1}{9})$ is not on the graph of g.

Ex: Linear Depreciation A printer has an original value of \$100,000 and is to be depreciated linearly over 5 years with a scrap value (see below) of \$30,000.

Does value depend on time? y depends on x

or does ~~time depend~~ on value?

Points look like: $(x, y)$
(time, value)

(a) Find an expression giving the book value at the end of year $t$.

$(5, 30{,}000)$
$(0, 100{,}000)$

Let $t$ = time in years
$V$ = value of printer in \$

$m = \dfrac{100{,}000 - 30{,}000}{0-5} = \dfrac{70{,}000}{-5} = -14{,}000$

$V - V_1 = m(t - t_1)$
$V - 100{,}000 = -14000(t - 0)$
$V - 100{,}000 = -14000t$

$$\boxed{V(t) = -14{,}000t + 100{,}000}$$

(b) What will be the book value at the end of year 2?

$V(2) = -14{,}000(2) + 100{,}000$
$= -28{,}000 + 100{,}000 = 72{,}000$

Value at end of year 2 is \$72,000.

$m = \dfrac{\Delta V}{\Delta t} = \dfrac{\text{how much value changed}}{\text{change in time}}$

(c) What is the rate of depreciation of the printer?

$m = \dfrac{\$\#}{\#\text{ years}}$

Rate of depreciation: \$14,000 per year

- Scrap value is the lowest value an asset obtains. Once an asset reaches scrap value, it remains at that value indefinitely. The scrap value is assumed to be zero, unless given other information regarding the actual scrap value or the time when scrap value is reached.

- Practical example of combination of functions: cost, revenue, and profit.
  - Fixed costs remain more or less constant regardless of a company's activity level. rent, salaries, utilities, insurance, taxes
  - Variable costs vary with production or sales. materials, utilities, salaries (overtime)
  - The total cost function is given by the sum of the variable and fixed costs.
  - The total profit function is given by the difference between the total revenue realized and the total cost incurred: $P(x) = R(x) - C(x)$

Revenue = amt of money business brings in

- Linear Cost, Revenue, Profit Functions
  - Let $x =$ the number of units of a product manufactured or sold.
  - Then $c =$the production cost of \$$c$ per unit (variable cost)
  - $F =$fixed costs in \$
  - $p =$ the selling price of \$$p$ per unit.
  - Cost Function: $C(x) = \underbrace{cx}_{\$} + \underbrace{F}_{\$}$
  - Revenue function: $R(x) = \overset{\$}{px}$
  - Profit function: $P(x) = R(x) - C(x)$

$P(x) = px - (cx + F)$

$P(x) = px - cx - F$

$\left(\frac{\$c}{\cancel{\text{per ball}}}\right)\left(\frac{\# \cancel{\text{balls}}}{1}\right) = \$$

$\left(\frac{\$p}{\text{ball}}\right)\left(\#\text{balls}\right) = \$$

- NOTE: Typically, total cost, revenue, and profit functions for a company will be nonlinear, but it is important that we examine the linear cases.

Ex: Malone Industries produces knitted accessories like scarves, shawls, and hats. For the division that produces knitted hats, each hat sells for \$12, and the variable cost of producing each unit is 30% of the selling price. The monthly fixed costs incurred by this division of Malone Industries are \$20,000.

(a) Find the cost, revenue, and profit functions.

$x = \#$ of hats

$\boxed{C(x) = 3.60x + 20{,}000}$

Cost to produce one hat: 30% of \$12

$.3(\$12) = \$3.60$ cost to make one hat

$R(x) = \left(\frac{\$12}{1\ \cancel{\text{hat}}}\right)\left(\frac{x\ \cancel{\text{hats}}}{1}\right) = \boxed{12x}$ measured in \$

$P(x) = R(x) - C(x) = 12x - (3.60x + 20{,}000)$

$= 12x - 3.60x - 20{,}000$

$\boxed{P(x) = 8.40x - 20{,}000}$

(b) What is the $x$-intercept for the profit function? What does it represent?

x-int: $P(x) = 0$

Solve for x: $0 = 8.40x - \cancel{20{,}000}$

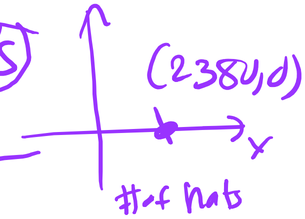
$+20{,}000 \qquad +\cancel{20{,}000}$

$\frac{20{,}000}{8.40} = \frac{\cancel{8.40}x}{\cancel{8.40}}$

PEMDAS

$x = 2380.952\ldots$

Profit is \$0 when 2380 hats are produced and sold.




linear C(x) points look like (x, C) (input, output)

Want $C(x)$ points look like $(x, C)$ (input, output)

output → input

Ex: It costs a company \$10,170 to produce 800 pairs of running shoes and \$13,810 to produce 1150 pairs. Find the cost function, and identify the fixed cost and unit (marginal) cost. Assume the model is linear.

$x$ = # of pairs of running shoes  $C$ is cost in dollars

$(x, C)$

$(800, 10170)$  $(1150, 13810)$  cost to make one pair!

$$m = \frac{13810 - 10170}{1150 - 800} = \frac{3640}{350} = \$10.40 \text{ per pair}$$

unit cost or variable cost or marginal cost

$$m = \text{rate of change} = \frac{\Delta \text{ cost in } \$}{\Delta \text{ \# of pairs}}$$

$C - C_1 = m(x - x_1)$

$C - 10{,}170 = 10.40(x - 800)$

$C - 10{,}170 = 10.40x - 8320$

$+10{,}170 \quad +10{,}170$

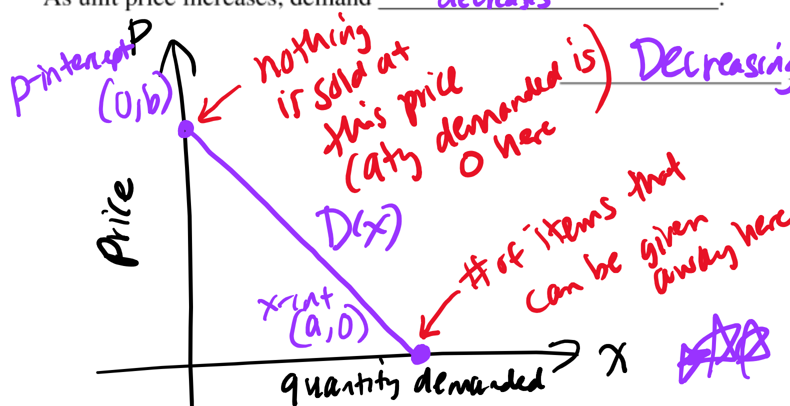
$C = 10.40x + 1850$

$\boxed{C(x) = 10.40x + 1850}$

Unit cost: \$10.40/pair
Fixed cost: \$1850

- Demand function Let $x$ be the number of units demanded and let $p$ be the price of each unit.
  Consumer demand for a commodity ~~depends on the commodity's unit price~~.
  As unit price decreases, demand increases.
  As unit price increases, demand decreases.

demand = ~~quantity~~ demanded




$D(x)$ is Decreasing function

Points look like $(x, p)$

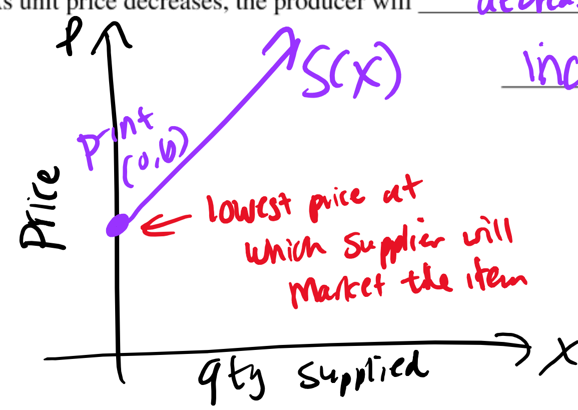
Slope is negative.

Consumer's point of view

Points look like $(x, p)$.  $p(x) = mx + b$  NOTE: Cannot have a negative number of items or a negative price.

- Supply function Let $x$ be the number of units supplied and let $p$ be the price of each unit.
  This is from the producer's/supplier's/company's point of view.
  As unit price increases, the producer will increase supply.
  As unit price decreases, the producer will decrease supply.

Points $(x, p)$




increasing function

from the supplier's point of view

Points look like $(x, p)$.  $p(x) = mx + b$  NOTE: Cannot have a negative number of items or a negative price.

Assume model is linear

Ex: A company makes rollerblades. When the unit price is \$180, quantity demanded is 10 pairs. Quantity demanded is 50 pairs when the price is \$100. The company is not willing to sell rollerblades for \$60 or less per pair. It will supply 10 pairs if it can get \$80 a pair.

$x$ = # of pairs of rollerblades

$p$ = price

$(x, p)$

(# of pairs, price in \$)

(a) Find the demand equation on your own. $p = D(x) = -2x + 200$

(b) Find the supply equation. key words: supply, company, supplier, producer, manufacturer

$(10, 80)$ $(0, 60)$

$m = \dfrac{60-80}{0-10} = \dfrac{-20}{-10} = 2 = \dfrac{+2}{+1}$

if price increases by \$2, supplier will supply 1 additional pair

$p - 60 = 2(x - 0)$

$p - 60 = 2x$

$p = 2x + 60$

$\boxed{S(x) = 2x + 60}$

$(x, p)$

(c) Above what price will there be no demand? $\Rightarrow$ use $D(x) = -2x + 200$

Find $p$ when $x = 0$

$p = -2x + 200$

$D(0) = p = -2(0) + 200$

$p = 200$

Above \$200, there will be no demand.

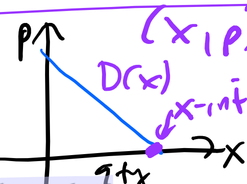
(d) What is the maximum quantity demanded? $\Rightarrow$ use $D(x) = -2x + 200$

Find $x$ (biggest $x$) $\Rightarrow$ look at $x$-int. $\Rightarrow p = 0$

$0 = -2x + 200$

$\dfrac{-200}{-2} = \dfrac{-2x}{-2} \Rightarrow x = 100$

Max. qty demanded is 100 pairs of rollerblades




(e) What is the lowest price at which the supplier will make rollerblades available in the market?

use $S(x) = 2x + 60$

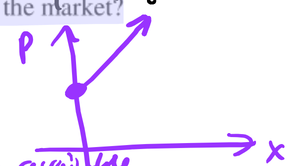
Find $p$ $x$-value is 0

look at p-int.

$S(0) = 2(0) + 60$

$p = S(0) = 60$

\$60 is lowest price they will make pairs available.




(f) How many pairs of rollerblades will consumers purchase if they are free?

use $D(x) = -2x + 200$

$x = ?$ when $p = 0$

$0 = -2x + 200$

$\dfrac{-200}{-2} = \dfrac{-2x}{-2}$

$x = 100$

Consumers will "purchase" 100 pairs if they are free

(g) Producers will only provide pairs of rollerblades if the price is above what value?

use $S(x) = 2x + 60$

Find $p$ $x = 0$ $\Rightarrow$

Producers will provide pairs only if price is above \$60.

(h) Above what price will consumers not buy any rollerblades?

$D(x) = -2x + 200$

$p = ?$ $x = 0$.

Consumers will not buy rollerblades at a price above \$200.