SECTION 2.3 SYSTEMS OF TWO EQUATIONS IN TWO UNKNOWNS

- Solutions to systems of linear equations

  Given any two lines L_1 and L_2 in the Cartesian plane, one and only one of the following may occur:

  - The lines L_1 and L_2 intersect at exactly one point. This describes an independent system.

    Ex: Solve $\begin{array}{l} 2x - y = 1 \\ 3x + 2y = 12 \end{array}$

$2(2x - y) = 2(1)$
$3x + 2y = 12$

$4x - \cancel{2y} = 2$
$3x + \cancel{2y} = 12$
$\frac{\cancel{7}x}{\cancel{7}} = 1$
$x = 2$

To find y-coordinate, plug x=2 into either eqn

$3(2) + 2y = 12$
$6 + 2y = 12$
$-6 \quad\quad -6$
$\frac{2y}{2} = \frac{6}{2}$
$y = 3$

Intersection point: $(2, 3)$

Conditional soln

$\boxed{\text{Exactly one soln}}$

  - L_1 and L_2 are parallel and coincident. This describes a dependent system.

    Ex: Solve $\begin{array}{l} 2x - 4y = 2 \\ 3x - 6y = 3 \end{array}$

$3(2x - 4y) = 3(2)$
$-2(3x - 6y) = -2(3)$

$6x - \cancel{12y} = \cancel{6}$
$\cancel{-6x} + \cancel{12y} = \cancel{-6}$
$0 = 0$ ✓

True, no matter what x or y are.

$\boxed{\text{Infinitely many solns.}}$

(x, y) $\quad (t, \frac{1}{2}t - \frac{1}{2})$

$2x - 4y = 2$
$\frac{-4y}{-4} = \frac{-2x}{-4} + \frac{2}{-4}$
$y = \frac{1}{2}x - \frac{1}{2}$

$3x - 6y = 3$
$\frac{-6y}{-6} = \frac{-3x}{-6} + \frac{3}{-6}$
$y = \frac{1}{2}x - \frac{1}{2}$

Let $x = t$, t is any real #

  - L_1 and L_2 are parallel and distinct. This describes an inconsistent system.

    Ex: Solve $\begin{array}{l} 3(5x + 20y) = 3(10) \\ 5(-3x - 12y) = 5(8) \end{array}$

$\boxed{\text{NO SOLN}}$

$\cancel{15x} + \cancel{60y} = 30$
$\cancel{-15x} - \cancel{60y} = 40$
$0 \neq 70$

False, no matter what x or y are.

$5x + 20y = 10$
$\frac{20y}{20} = \frac{-5x}{20} + \frac{10}{20}$
$y = -\frac{1}{4}x + \frac{1}{2}$

Same slope, but different y-int.

$-3x - 12y = 8$
$\frac{-12y}{-12} = \frac{3x}{-12} + \frac{8}{-12}$
$y = -\frac{1}{4}x - \frac{2}{3}$

- Given two nonparallel lines L_1 and L_2 ($L1 : y = m_1x + b_1, L2 : y = m_2x + b_2$), we can find their point of intersection.

  Ex: Find the intersection point of the lines $y = x + 1$ and $y = -2x + 4$.

$y = y$

$x + 1 = -2x + 4$

$+2x \qquad +2x$

$3x + 1 = 4$

$-1 \qquad -1$

$\frac{3x}{3} = \frac{3}{3}$

$x = 1$

Plug $x=1$ into either eqn:

$y = 1 + 1$

$y = 2$

$(1,2)$ exactly one soln.

$P(x) = R(x) - C(x)$

$R(x) = C(x)$

$R(x) - C(x) = 0$

$P(x) = 0$

- Break even levels of operation:

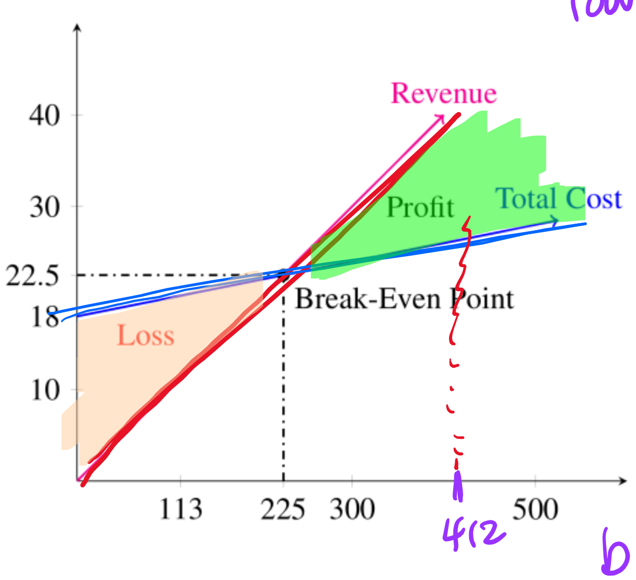
  When revenue received from selling x items exceeds cost, the company makes a profit

  When cost exceeds revenue, the company takes a loss.

- Break even point: the point where the company neither makes a profit nor takes a loss.

  This point is the intersection of the cost and revenue functions.

(# of units, $ in 1000's)



$b = 0.$

Consider the graph:

(a) How many units must the company produce to break even? What is the revenue earned when the company breaks even?

- 225 units
- Revenue earned is $22,500

(b) If the company produces and sells 412 units, is a loss incurred or profit earned?

(c) Find the revenue function from the graph.

points: $(225, 22.5)$ & $(0,0)$

$m = \frac{22.5 - 0}{225 - 0} = \frac{22.5}{225} = \frac{1}{10} = .1$

$R(x) = \frac{1}{10}x$

or $R(x) = .1x$ or $R(x) = .10x$

Ex: Revisiting a previous example: Malone Industries produces knitted accessories like scarves, shawls, and hats. For the division that produces knitted hats, each hat sells for \$12, and the variable cost of producing each unit is 30% of the selling price. The monthly fixed costs incurred by this division of Malone Industries are \$20,000.

(a) What is the break-even point for the division?

Recall: $x = \#$ of hats

$C(x) = 3.60x + 20{,}000$

$R(x) = 12x$

$P(x) = 12x - (3.60x + 20{,}000)$

To find y-coord: $R(2380.95) \approx$

$C(x) = R(x)$

$3.60x + 20{,}000 = 12x$

$\frac{20{,}000}{8.40} = \frac{8.40x}{8.40}$

$x \approx 2380.95$

(2380 hats, \$28560)

(2380 hats, \$28,571.40)

plugged in 2380.95

(b) Do you notice any relationship between the x-intercept for the profit function (see Section 2.2) and the break-even quantity?

SAME!

$P(x) = 0$ is the same as $C(x) = R(x)$

(c) What is the loss sustained by the division if only 1200 units are produced and sold each month?

use profit fcn

Recall: $P(x) = 8.40x - 20{,}000$

$x = 1200$

Find P

$P(1200) = 8.4(1200) - 20{,}000$

$= -9920$

Loss of \$9920.

(d) What is the profit earned by the division if 3500 hats are produced and sold each month?

$P(3500) = 8.40(3500) - 20{,}000$

$= \$9400$

$x = 3500$ Find P

Profit of \$9400

(e) Suppose the division of Malone Industries that makes shawls has monthly fixed costs of \$35,000. Each shawl costs \$15 to make and sells for \$48. Find the break-even point for this division of the company.

$x = \#$ of shawls (x, p)

$C(x) = 15x + 35000$

$R(x) = 48x$

Break-even point: $C(x) = R(x)$

$15x + 35000 = 48x$

$-15x \quad -15x$

$\frac{35000}{33} = \frac{33x}{33}$

$x = 1060.606060\ldots$

use calculator:

48 $\boxed{\times}$ $\boxed{2nd}$ $\boxed{Ans}$

$R(1060.6060\ldots)$

$= 50909.0909\ldots$

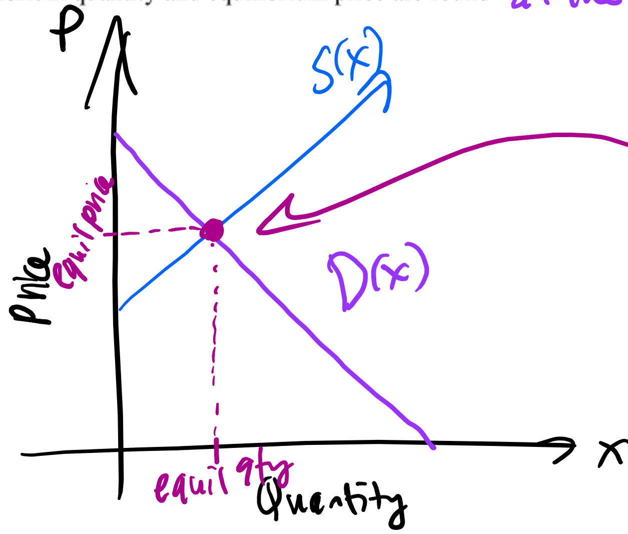
Break-even point:

(1060 shawls, \$50,909.09)

1061 shawls

demand = supply

- Market equilibrium: Achieved when quantity demanded = qty supplied
  Equilibrium quantity and equilibrium price are found at the intersection of supply & demand curves.



(x, p)
equil point
(equil qty, equil price)

Ex: Revisiting a previous example (from Section 2.2): A company makes rollerblades. When the unit price is \$180, quantity demanded is 10 pairs. Quantity demanded is 50 pairs when the price is \$100. The company is not willing to sell rollerblades for \$60 or less per pair. It will supply 10 pairs if it can get \$80 a pair. What is the equilibrium point for the sale of rollerblades?

x = # of pairs of rollerblades

p = \$

(x, p)

RECALL FROM 2.2:
$D(x) = -2x + 200$ and $S(x) = 2x + 60$

$p = -2x + 200 \qquad p = 2x + 60$

$p = p \iff D(x) = S(x)$

$-2x + 200 = 2\cancel{x} + 60$

$-2x \qquad -2x$

$-4x + 200 = 60$

$-200 \qquad -200$

$-4x = -140$

$\div -4 \qquad \div -4$

$x = +35$

Plug $x = 35$ into $D(x)$ or $S(x)$

$D(35) = -2(35) + 200$

$= -70 + 200$

p when $x = 35$

$p = 130$

Equilibrium point: (35 pairs, \$130)

equil qty: 35 pairs

equil price: \$130

p $D(x) = -2x + 200$

input