

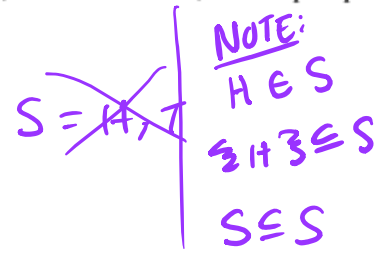
## MATH 1324 – FINITE MATHEMATICS SECTION 4.1 MATHEMATICAL EXPERIMENTS

- An experiment is an activity with observable results.
- A random experiment is an activity or an observation whose results cannot be predicted ahead of time. The following are random experiments:
  - Tossing a coin and observing if it is heads or tails
  - Casting a die and observing the number on top
  - Testing a spark plug from a batch of 100 and observing whether or not it is defective.
- The results of experiments are called outcomes or sample points.
- A sample space is the set consisting of all possible outcomes of the experiment. Denoted by  $S$
- An event is a subset of the sample space (a collection of outcomes from an experiment).  
NOTE:  $2^{\# \text{ of outcomes}} = \text{total } \# \text{ of possible events}$
- An event which consists of exactly one outcome is called a simple event of the experiment.
- An impossible event is an event containing no outcomes. Denoted by  $\emptyset$  or  $\{\}$

Ex: For the experiment of tossing a coin and observing heads or tails, list the outcomes, the sample space, and the events.

- Outcomes: Heads  $\rightarrow H$  or Tails  $\rightarrow T$
  - Sample space:  $S = \{H, T\}$
  - Events:  $\emptyset, \{H\}, \{T\}, \{H, T\}$
- NOTE: Unless stated otherwise, when we say die, we are referring to a standard six-sided die.

*We will have  
 $2^2 = 4$  events*



Ex: Experiment: Roll a fair die and observe the number on top. Give the sample space and simple events.  
 $S = \{1, 2, 3, 4, 5, 6\}$

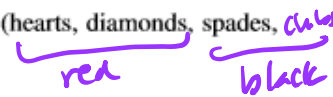
Simple events:

- $\{1\}, \{2\}, \{3\}, \{4\}, \{5\}, \{6\}$

*NOTE: there are  $2^6 = 64$  total events*

Ex: Experiment: A card is selected at random from a standard 52 card deck, and its suit (hearts, diamonds, spades, clubs) is recorded. Give the sample space and simple events.

$S = \{\heartsuit, \diamondsuit, \spadesuit, \clubsuit\}$



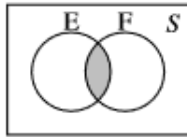
Simple events:

- $\{\heartsuit\}, \{\diamondsuit\}, \{\spadesuit\}, \{\clubsuit\}$

$2-10, J, Q, K, A$

*NOTE: Total events =  $2^4 = 16$*

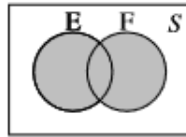
Intersection of events  $E$  and  $F$ :



The event that contains all outcomes in both  $E$  and  $F$ .

$$E \cap F = \{x \mid x \in E \text{ and } x \in F\}$$

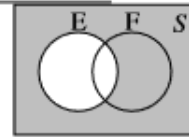
Union of events  $E$  and  $F$ :



The event that contains all outcomes that belong to either  $E$  or  $F$  or both.

$$E \cup F = \{x \mid x \in E, x \in F, x \in E \cap F\}$$

Complement of event  $E$ :



The event that contains all outcomes in  $S$  but not in  $E$ .

$$E^c = \{x \mid x \in S \text{ and } x \notin E\}$$

Mutually exclusive:



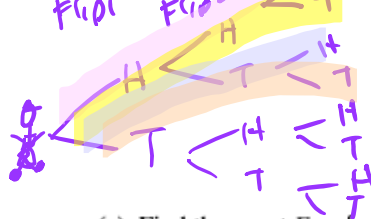
Events  $E$  and  $F$  have no outcomes in common.

$$E \cap F = \emptyset$$

Ex: Let  $S = \{1, 2, 3, 4, 5, 6\}$ ,  $E = \{2, 4, 6\}$ ,  $F = \{1, 3, 5\}$ , and  $G = \{5, 6\}$ . Find the event  $(E \cap F \cap G)^c$ .

$E \cap F = \emptyset$  so  $E \cap F \cap G = \emptyset$  so  $(E \cap F \cap G)^c = \emptyset^c = S = \{1, 2, 3, 4, 5, 6\}$

Ex: Suppose our experiment is to flip a fair coin 3 times. Give  $S$ .



$S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$

$\frac{2}{\text{Flip 1}} \cdot \frac{2}{\text{Flip 2}} \cdot \frac{2}{\text{Flip 3}} = 8 \text{ outcomes}$

(a) Find the event  $E = \{x \mid x \text{ has exactly one head}\}$

$E = \{HTT, THT, TTH\}$

(b) Find the event  $F = \{x \mid x \text{ has two or more heads}\}$

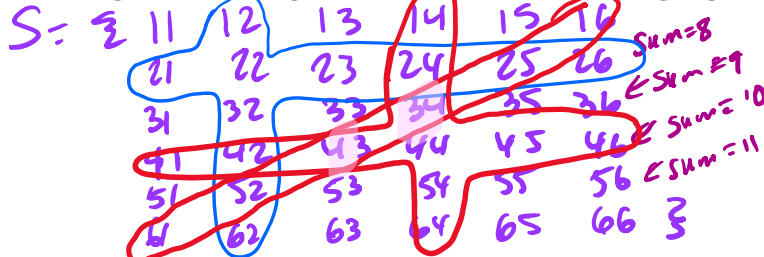
$F = \{HHH, HHT, HTH, THH\}$

(c) Find the event  $G = \{x \mid x \text{ has more than three heads}\}$

$G = \emptyset$

Ex: Experiment: Roll a six-sided die. Give the sample space:  $S = \{1, 2, 3, 4, 5, 6\}$

Ex: Experiment: Roll a pair of six-sided dice. Give the sample space.



(a) Find the event  $E = \{\text{the sum is } 8\}$

$E = \{26, 35, 44, 53, 62\}$

(b) Find the event  $F = \{\text{one die is a } 2\}$

$F = \{21, 22, 23, 24, 25, 26, 12, 32, 42, 52, 62\}$

(c) Find the event  $G = \{\text{one die is at least a } 5\}$  5 or 6

$\{51, 52, 53, 54, 55, 56, 61, 62, 63, 64, 65, 66, 15, 25, 35, 45, 16, 26, 36, 46\}$

(d) Find the event  $H = \{\text{the sum is } 7 \text{ or one die is a } 4\}$

$\{14, 24, 34, 44, 54, 64, 41, 42, 43, 45, 46, 16, 25, 52, 61\}$

(e) Find the event  $J = \{\text{the sum is } 7 \text{ and one die is a } 4\}$

$J = \{34, 43\}$