MATH 1324 – FINITE MATHEMATICS
SECTION 4.2 BASICS OF PROBABILITY

- Suppose we repeat an experiment n times and event E occurs m of those times. Then $\frac{m}{n}$ is called the relative frequency of event E.
- Often, the more we repeat an experiment, the more the relative frequency approaches a certain value. This value, the empirical probability, is the measure of the amount of time that the event will occur in the long run.
- The probability of an event is a number between 0 and 1 that represents the likelihood of that event occurring. The larger the probability, the more likely the event is to occur.

Ex: Coin T, T, H, T, T, H, T, H

Prob = favorable / total

Ex: A survey is done of people making purchases at a gas station. Most people buy gas or a drink. The results are shown in the table below.

	buy drink (D)	no drink (D^c)	TOTAL
buy gas (G)	20	15	35
no gas (G^c)	10	5	15
TOTAL	30	20	50

$n(G \cup D) = n(G) + n(D) - n(G \cap D)$
$= 35 + 30 - 20$

(a) What is the probability someone at this gas station buys a drink?

$P(D) = \frac{30}{50} = \frac{3}{5} = .6$

Prob = favorable / total

(b) What is the probability someone at this gas station buys gas ∩ and a drink?

$P(G \cap D) = \frac{20}{50} = \frac{2}{5} = .4$

(c) What is the probability someone at this gas station does not buy gas?

$P(G^c) = \frac{15}{50}$

(d) What is the probability someone at this gas station buys gas ∪ or buys a drink?

$P(G \cup D) = \frac{35 + 30 - 20}{50} = \frac{45}{50}$

- A probability distribution for an experiment is a table of all the possible outcomes and their corresponding probabilities.
- In order for a probability distribution with n outcomes $(x_1, \ldots, x_n)$ to be valid, it must satisfy the following conditions:

1. $0 \le P(s_i) \le 1$ for $i = 1, 2, 3, \ldots n$ — Each probability is a number between 0 and 1.
2. $P(s_1) + P(s_2) + P(s_3) + P(s_4) + \ldots + P(s_n) = 1$ — All simple probabilities add up to 1.

Fair die: $S = \{1, 2, 3, 4, 5, 6\}$ $P(1) = P(2) = P(3) = P(4) = P(5) = P(6) = \frac{1}{6}$

- Sample spaces in which the outcomes are equally likely are called uniform sample spaces.
- So, for a uniform sample space $S = \{s_1, s_2, s_3, \ldots, s_n\}$, we can assign probabilities to the simple events:

$P(S) = 1$ so $P(s_1) = P(s_2) = P(s_3) = P(s_4) = \ldots = P(s_n) = \frac{1}{n}$

- NOTE: $P(\varnothing) = 0$

Ex: Experiment: A card is selected at random from a standard 52 card deck, and its suit (hearts, diamonds, spades, clubs) is recorded. Fill in the probability distribution.
RECALL: $S = \{\heartsuit, \clubsuit, \diamondsuit, \spadesuit\}$

Outcome	♡	♣	◇	♠
Probability	$\frac{13}{52}$	$\frac{13}{52}$	$\frac{13}{52}$	$\frac{13}{52}$

Is this a uniform sample space? Yes
all outcomes are equally likely.

Ex: Grade distributions are shown in the table. Determine the probability distribution associated with this data.

Grade distribution

Grade	A	B	C	D	F
Frequency of Occurrence	4	12	22	17	3

Probability distribution

Grade	A	B	C	D	F
Probability	$\frac{4}{58}$	$\frac{12}{58}$	$\frac{22}{58}$	$\frac{17}{58}$	$\frac{3}{58}$

Is this a uniform sample space? NO $4+12+22+17+3 = 58$

Ex: If a ball is selected at random from a box containing 3 red, 4 yellow, and 8 green balls, what is the probability the ball will be white? green?

$P(\text{white}) = \frac{0}{3+4+8} = 0$ $P(\text{green}) = \frac{8}{3+4+8} = \frac{8}{15}$ $\frac{\text{favorable}}{\text{total}}$

Ex: In a sweepstakes, 100,000 entries have been received. If 1 grand prize, 5 first prizes, 25 second prizes, and 500 third prizes are to be awarded, what is the probability that someone who submitted one entry will win

- the grand prize? $P(\text{grand prize}) = \frac{1}{100{,}000}$
- any prize? $P(\text{any prize}) = \frac{531}{100{,}000}$

Ex: Of 183 people employed at a software company, 23% are left-handed. If a single employee is chosen at random, what is the probability the employee is left-handed?

$P(\text{left-handed}) = .23$ $.23(183) = 42.09$

$\frac{42}{183} \approx 0.2295081$

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