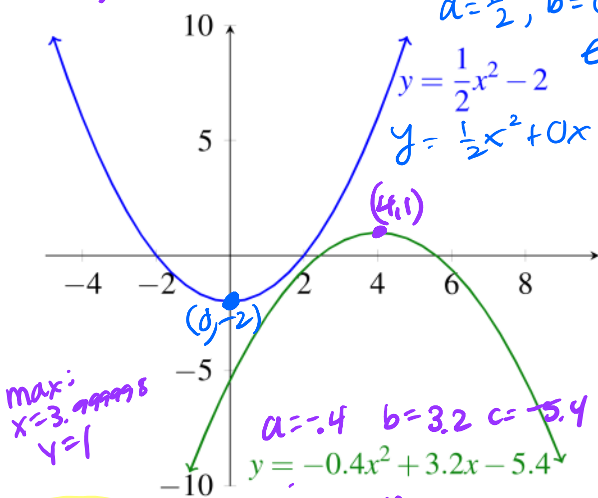
SECTION 5.1-5.3 FUNCTIONS/POLYNOMIAL FUNCTIONS/RATIONAL FUNCTIONS

- A quadratic function is a second-degree polynomial function in one variable. Looks like $a \neq 0$

vertex $\left(\frac{-b}{2a}, y\right)$

Form	Vertex	Axis of Symmetry	If $a > 0$	If $a < 0$
$f(x) = ax^2 + bx + c$	$\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$	$x = \frac{-b}{2a}$	+ +	− −
$f(x) = a(x-h)^2 + k$	(h, k)	$x = h$		



$y = \frac{1}{2}x^2 + 0x - 2$

$a = \frac{1}{2}$, $b = 0$, $c = -2$ ← Vertex: x-coordinate: $x = \frac{-b}{2a}$, $x = \frac{-0}{2(\frac{1}{2})} = 0$; y-coordinate: $y = \frac{1}{2}(0)^2 - 2$, $y = -2$

(0, -2)

- What is the minimum value: -2 (min y-value)
- What is the maximum? None
- Where is the minimum? (0, -2)

max: $x = 3.99998$, $y = 1$

$a = -.4$ $b = 3.2$ $c = -5.4$ in purple

Vertex: $x = \frac{-b}{2a} = \frac{-3.2}{2(-.4)} = +4$

y-coord: $y = -.4(4)^2 + 3.2(4) - 5.4 = 1$ (4,1)

- What is the min? None!
- What is the max? 1
- Where is the max? (4,1)

- RECALL: We can write revenue as $R(x) = px$ if the item being sold had a fixed selling price, p. (x, p)
Also the selling price of an item could be $p(x) = mx + b$, (linear price-demand function).
⋆⋆ If you are given $p(x)$, then the revenue function is given by $R(x) = p \cdot x = (mx+b)x = mx^2 + bx$. ⋆⋆

Revenue fcn = (Demand fcn)(x)

Ex: The total cost, in dollars, to produce x "Math is Awesome" T-shirts is $C(x) = 2x + 26$ for $x \geq 0$. The price-demand function, in dollars per shirt, is given by $p(x) = 30 - 2x$; for $0 \leq x \leq 15$. x = # of t-shirts

(demand fcn)(x) ← input is x

demand (x, p)

(a) Find the revenue function, $R(x)$.

$R(x) = px = (30 - 2x)x = x(30 - 2x) = 30x - 2x^2$

input is x   multiply by x

$\boxed{R(x) = -2x^2 + 30x}$

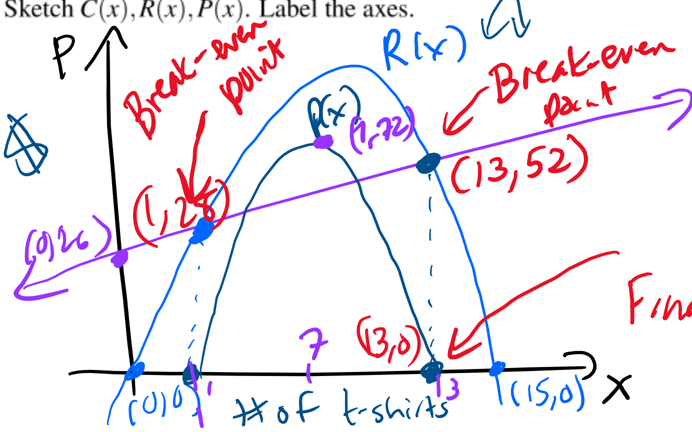
Revenue (# of items, $)

(b) Find the profit function, $P(x)$.

$P(x) = R(x) - C(x) = (30x - 2x^2) - (2x + 26)$ $C(x) = 2x + 26$

$= 30x - 2x^2 - 2x - 26 = \boxed{-2x^2 + 28x - 26 = P(x)}$

(c) Sketch $C(x), R(x), P(x)$. Label the axes.

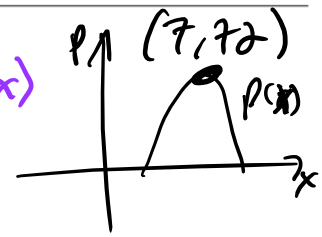


If Profit = 0, cost = revenue

Find where P(x) = 0

(x, p) Profit: (# of Shirts, $)

(d) Find the number of T-shirts which need to be sold in order to maximize profit.



Find x-value at highest P value (vertex)

need to sell 7 t-shirts to maximize profit

(e) Find the maximum profit.

max profit is $72

We know maximum profit occurs at $x=7$

(f) Find the price to charge per T-shirt, in order to maximize profit. Watch the units! use demand (x, $ for 1 shirt)

$p(x)=30-2x$ Plug in the x-value where profit was maximized

$p(7)=30-2(7)$
$=30-14=16$

Should charge $16 per shirt.

(g) Interpret the break-even quantity/quantities, if any exist.

• What is the revenue at $16 per shirt?
$R(x)=x(\text{demand})$
or what is the revenue when profit is maximized?
$R(7)=7(16)$

$C(x)=R(x)$ or $P(x)=0$
and calc intersect 2nd calc zero
$x=1$ or $x=13$
If 1 shirt or 13 shirts are sold, no profit and no loss.

Ex: Revisiting a previous example (from Section 2.2): A company makes rollerblades. When the unit price is $180, quantity demanded is 10 pairs. Quantity demanded is 50 pairs when the price is $100. The company is not willing to sell rollerblades for $60 or less per pair. It will supply 10 pairs if it can get $80 a pair.

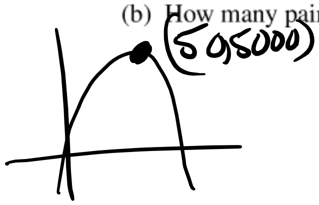
☆ How is revenue related to demand? ☆
$R(x)=x(\text{demand fcn})$ ☆
(# of items)($ per item) = $

RECALL FROM 2.2:
$D(x)=-2x+200$ and $S(x)=2x+60$ ☆

(a) Find the revenue function, $R(x)$.

$R(x)=x(-2x+200)$
$R(x)=-2x^2+200x$

(b) How many pairs of rollerblades should be sold to maximize revenue? What is the maximum revenue?



Should sell 50 pairs to maximize revenue.

Max revenue is $5000

(c) At what price should pairs of rollerblades be sold in order to maximize revenue?

revenue.

(c) At what price should pairs of rollerblades be sold in order to maximize revenue?

Find p $\Rightarrow$ demand fcn: $p = -2x + 200$

$= -2(50) + 200$

$= -100 + 200 = 100$

Should sell each pair for \$100

- If the total cost is $C(x)$, then the average cost function is $\bar{C}(x) = \dfrac{C(x)}{x}$, which is the average cost per item for x items produced.

$x = \#$ of items produced

- The average cost function is a rational function, which is a polynomial divided by a polynomial.

Ex: If the total cost function for a commodity is $C(x) = 0.1x^2 + 50x + 90$, where x gives the number of items produced,

(a) find the average cost, $\bar{C}$.

$$\bar{C}(x) = \frac{C(x)}{x} = \frac{.1x^2 + 50 + 90}{x}$$

$$= \frac{.1x^2}{x} + \frac{50\cancel{x}}{\cancel{x}} + \frac{90}{x}$$

$$\boxed{\bar{C}(x) = .1x + 50 + \frac{90}{x}}$$

Scratch:

$$\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b}$$

$$\frac{.1x^2}{x} = \frac{.1x\cancel{x}}{\cancel{x}}$$

$(x, \bar{C})$

(#of items, average cost)

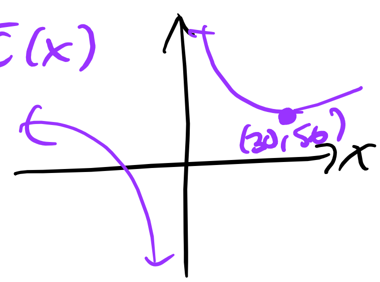
use y-value from calc.

(b) find the minimum average cost.

Graph average cost $\bar{C}(x)$

2nd Calc

3: minimum



Minimum average cost is \$56

occurs when 30 items are sold

$$Ax + By = C$$
$$y = mx + b$$

lines: x & y are raised to the 1st power.

no roots
no fractional powers

$x = 2$ $\updownarrow$ $y = 7$ $\leftrightarrow$

$3x + 4y = 19$ $y = -7x + 2$

Parabolas:

- Highest power of x is 2.
  y is to the 1st power

$y = +x^2 + \underline{\quad}$
all x stuff

$y = -x^2 + \underline{\quad}$
all x stuff