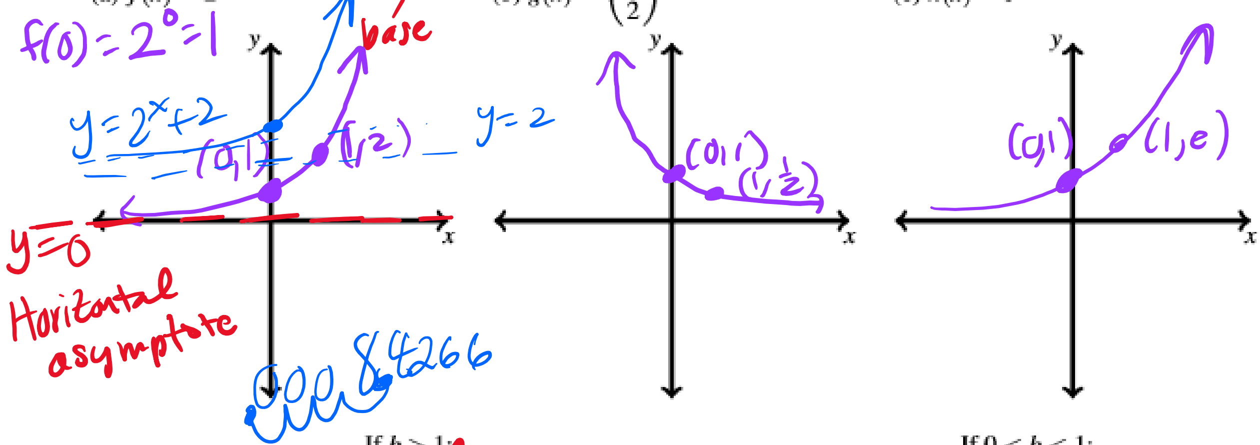
MATH 1324 – FINITE MATHEMATICS
SECTION 5.6-5.8 EXPONENTIAL & LOGARITHMIC FUNCTIONS/TRANSFORMATIONS

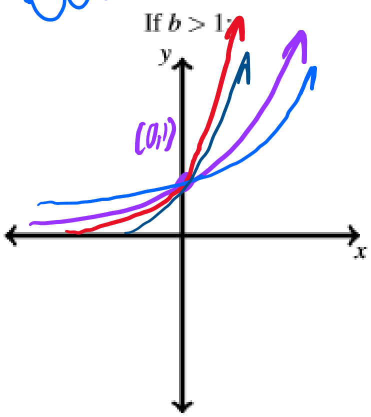
Ex: Graph

$y = b^x \leftarrow$ power/exponent

(a) $f(x) = 2^x$ (b) $g(x) = \left(\frac{1}{2}\right)^x$ (c) $h(x) = e^x$

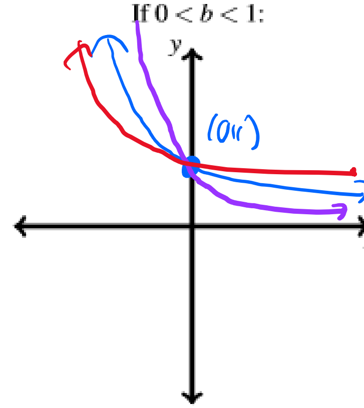


If $b > 1$:



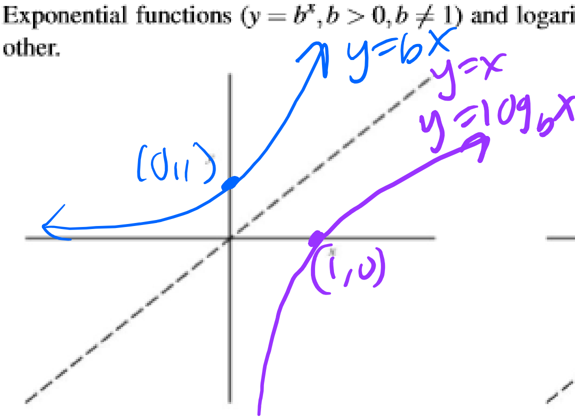
Growth

If $0 < b < 1$:

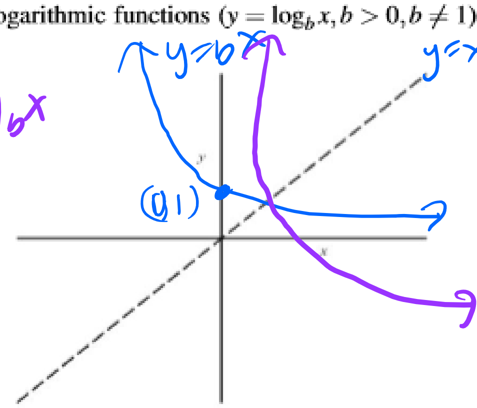


Decay

- We can also shift and reflect graphs of $f(x) = b^x$. Sketch $y = 2^x + 2$ with the graph of $y = 2^x$.
- $e \approx 2.71812818284\ldots$
- Exponential functions ($y = b^x, b > 0, b \neq 1$) and logarithmic functions ($y = \log_b x, b > 0, b \neq 1$) are inverses of each other.



$y = b^x$ and $y = \log_b x$
$b \neq 1$ and $b > 1$



$y = b^x$ and $y = \log_b x$
$b \neq 1$ and $0 < b < 1$

- NOTE: $\log x = \log_{10} x$ and $\ln x = \log_e x$.
- Exponential and/or logarithmic functions, sometimes combined with polynomial or other functions, are often used to model behavior of real world phenomenon.
- Financial applications are modeled with exponentials functions. We will discuss those in Chapter 6.

Ex: The sound intensity (β) level, measured in decibels, is given by $\beta = 10\log\left(\frac{I}{I_0}\right)$ where $I_0 = 10^{-12}$ watts per square meter is the reference intensity and I, measured in watts per square meter, is the sound intensity.

(a) If the sound intensity of a siren from a fire truck is 2 watts per square meter, find the sound intensity level β.

$I = 2 \frac{\text{watts}}{m^2}$ Find β

$\beta = 10\log\left(\frac{2}{10^{-12}}\right)$ $10^{-12} \rightarrow$ Type 10^(-12) or 1 EE-12

$\beta \approx 123.0103$ decibels

(b) If the city fireworks show has sound intensity level of $\beta = 142$, find the sound intensity.

$\beta = 10\log\left(\frac{I}{10^{-12}}\right)$ Find I

$\underbrace{142}_{Y1} = \underbrace{10\log\left(\frac{I}{10^{-12}}\right)}_{Y2}$ $I \approx 158.5 \frac{\text{watts}}{m^2}$

(c) Compare sound intensities of 123 decibels to 142 decibels.

$142 - 123 = 19$ decibels difference

123 decibels $\Rightarrow I = 2$ watts/m2
142 decibels $\Rightarrow I = 158.5$ watts/m2

$\frac{\text{bigger}}{\text{smaller}} = \frac{158.5}{2} = 79.25$

Fireworks are 79.25 times more intense than the firetruck siren.

Ex: A surge function can be used to represent the way a drug interacts in the bloodstream. Studying this function is essential to doctors and pharmacists because it allows them to administer dosages of medicine correctly. Consider this surge function giving concentration C in the patient's bloodstream, measured in ng/mL (nanograms per milliter), of a particular medication at time t, given in hours: $C(t) = 6.2t^4e^{-0.5t}$

Tarko, Olta (2021) "Surge Functions and Drug Interactions," Undergraduate Journal of Mathematical Modeling: One + Two: Vol. 12: Iss. 1, Article 7. Available at: https://digitalcommons.usf.edu/ujmm/vol12/iss1/7

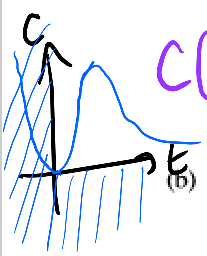
$C(t)$ input

(t, C) (time in hrs, conc. in ng/mL)

(a) Find the concentration of the medication after 90 minutes.

$\frac{90 \text{ minutes}}{} \cdot \frac{1 \text{ hr}}{60 \text{ minutes}}$

Find C when $t = \frac{90}{60}$



$C\left(\frac{90}{60}\right) = C(1.5) = 6.2(1.5)^4 e^{-.5(1.5)} \approx 14.826$ ng/mL

(b) Find the maximum concentration of the medication in the patient's bloodstream. At what time is the concentration a maximum?

- Max concentration: 465.13 ng/mL
- Max conc occurs after 8 hrs.

Max on Calc: (8.0000038, 465.12931)
(8, 465.12931)

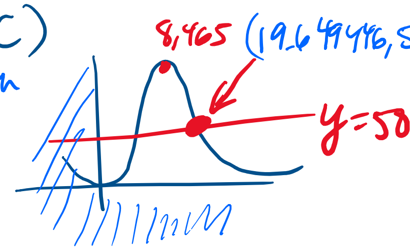
(c) If a second dose can be administered once the drug concentration has reached a minimum effectiveness of 50 ng/mL, when should a second dose be administered?

$C = 50$ ng/mL Find t (t, C)

$\underbrace{50}_{Y2} = \underbrace{6.2t^4e^{-.5t}}_{Y1}$

Round to nearest tenth of an hour.

Give 2nd dose at 19.6 hrs.



- NEWTON'S LAW OF COOLING: The temperature T of an object at time t is given by $T(t) = T_a + (T_0 - T_a)e^{-kt}$, where T_a is the ambient temperature (temperature of the surroundings), T_0 is the initial temperature, and k is an experimentally-determined constant. (NOTE: k is different depending on the object. $k > 0$)

  Ex: Suppose a body has been discovered and homicide detectives have a suspect in custody. The suspect has an alibi after 7pm the previous night. At 7am, at the crime scene, the body temperature was 84.3° and the air temperature was 78°. If the normal human body temperature is 98.6° and $k = 0.1947$, does the alibi rule out the suspect?

video quiz

$a^0 = 1 \qquad (a \neq 0)$

$x^0 = 1 \qquad (x \neq 0)$

$\cancel{\frac{x^3}{x^3}} = 1 \qquad \frac{x^3}{x^3} = x^{3-3} = x^0$