Name:_____ Math 2412-YOSKO Exploring Pascal's Triangle

SHOW ALL WORK AND JUSTIFY ALL ANSWERS.

1. Expand $(2x-y)^2 =$

2. Expand $(2x-y)^3 = (2x-y)(2x-y)^2 = (2x-y)(4x^2-4xy+y^2) =$

3. Expand $(2x-y)^4 = (2x-y)(2x-y)^3 = (2x-y)(8x^3-12x^2y+6xy^2-y^3)$
   HINT: Don't expand in a line. Line up like terms:

	x^4 terms	x^3y terms	x^2y^2terms	xy^3terms	y^4terms	
$(2x-y)(8x^3-12x^2y+6xy^2-y^3) =$	$16x^4$	$-24x^3y$	$+12x^2y^2$	$-2xy^3$		multiply $2x$ by terms in 2nd polynomial
		$-8x^3y$	$+12x^2y^2$	$-6xy^3$	$+y^4$	multiply $-y$ by terms in 2nd polynomial
	$16x^4$	$-32x^3y$	$+24x^2y^2$	$-8xy^3$	$+y^4$	

   Thus, $(2x-y)(8x^3-12x^2y+6xy^2-y^3) = 16x^4-32x^3y+24x^2y^2-8xy^3+y^4$

4. Expand $(2x-y)^5 = (2x-y)(2x-y)^4 = (2x-y)(16x^4-32x^3y+24x^2y^2-8xy^3+y^4)$

	x^5 terms	x^4y terms	x^3y^2terms	x^2y^3terms	xy^4terms	y^5terms
$(2x-y)(16x^4-32x^3y+24x^2y^2-8xy^3+y^4) =$	$32x^5$					
fill in these rows:						$-y^5$

   $(2x-y)(16x^4-32x^3y+24x^2y^2-8xy^3+y^4) =$

5. Expand $(2x-y)^6 = (2x-y)(2x-y)^5 = (2x-y)(32x^5-80x^4y+80x^3y^2-40x^2y^3+10xy^4-y^5)$
   x^6 terms x^5y terms x^4y^2terms x^3y^3terms x^2y^4terms xy^5terms y^6terms
   =

   _____

   $(2x-y)^6 =$

   Let's look at these binomial expansions.

$$\begin{aligned}
(2x-y)^2 &= 4x^2-4xy+y^2\\
(2x-y)^3 &= 8x^3-12x^2y+6xy^2-y^3\\
(2x-y)^4 &= 16x^4-32x^3y+24x^2y^2-8xy^3+y^4\\
(2x-y)^5 &= 32x^5-80x^4y+80x^3y^2-40x^2y^3+10xy^4-y^5\\
(2x-y)^6 &= 64x^6-192x^5y+240x^4y^2-160x^3y^3+60x^2y^4-12xy^5+y^6
\end{aligned}$$

6. What do you notice about the power of x as we start with the leading term and then move from term to term, from left to right?

7. What do you notice about the power of y as we start with the leading term and then move from term to term, from left to right?

   Let's write these expansions another way:

$$\begin{aligned}
(2x-y)^0 &= 1\\
(2x-y)^1 &= 1(2x)^1+1(-y)^1\\
(2x-y)^2 &= 1(2x)^2+2(2x)(-y)+1(-y)^2\\
(2x-y)^3 &= 1(2x)^3+3(2x)^2(-y)+3(2x)(-y)^2+1(-y)^3\\
(2x-y)^4 &= 1(2x)^4+4(2x)^3(-y)+6(2x)^2(-y)^2+4(2x)(-y)^3+1(-y)^4\\
(2x-y)^5 &= 1(2x)^5+5(2x)^4(-y)+10(2x)^3(-y)^2+10(2x)^2(-y)^3+5(2x)(-y)^4+1(-y)^5\\
(2x-y)^6 &= 1(2x)^6+6(2x)^5(-y)+15(2x)^4(-y)^2+20(2x)^3(-y)^3+15(2x)^2(-y)^4+6(2x)(-y)^5+1(-y)^6
\end{aligned}$$

8. What do you notice about the power of $2x$ as we start with the leading term and then move from term to term, from left to right?

9. What do you notice about the power of $-y$ as we start with the leading term and then move from term to term, from left to right?

Let's look at $(A+B)$ and consider those binomial expansions:

$$\begin{aligned}
(A+B)^0 &= 1\\
(A+B)^1 &= 1(A)^1+1(B)^1\\
(A+B)^2 &= 1(A)^2+2(A)(B)+1(B)^2\\
(A+B)^3 &= 1(A)^3+3(A)^2(B)+3(A)(B)^2+1(B)^3\\
(A+B)^4 &= 1(A)^4+4(A)^3(B)+6(A)^2(B)^2+4(A)(B)^3+1(B)^4\\
(A+B)^5 &= 1(A)^5+5(A)^4(B)+10(A)^3(B)^2+10(A)^2(B)^3+5(A)(B)^4+1(B)^5\\
(A+B)^6 &= 1(A)^6+6(A)^5(B)+15(A)^4(B)^2+20(A)^3(B)^3+15(A)^2(B)^4+6(A)(B)^5+1(B)^6
\end{aligned}$$

Let's look at just the coefficients of the terms of each binomial expansion. This is called Pascal's Triangle:

Power= 0	1
Power= 1	1 1
Power= 2	1 2 1
Power= 3	1 3 3 1
Power= 4	1 4 6 4 1
Power= 5	1 5 10 10 5 1
Power= 6	1 6 15 20 15 6 1
Power= 7	
Power= 8	
Power= 9	

Can you figure out what the coefficients are for binomial expansions with powers of 7, 8, and 9?
HINT: There are ones at the beginning and end of each row. Then, for each row, add two adjacent numbers. That sum goes beneath those numbers in the next row.

10. Expand $(2x^2y+3xy^2)^5$ using Pascal's Triangle.

Identify the coefficients of each term from Pascal's Triangle: 1 5 _ _ 5 1

The 1st term in the binomial is: _____ The 2nd term in the binomial is: _____

The expansion will be:

$$(\text{1st term}+\text{2nd term})^5 = 1(\text{1st})^5+5(\text{1st})^4(\text{2nd})+10(\text{1st})^3(\text{2nd})^2+10(\text{1st})^2(\text{2nd})^3+5(\text{1st})(\text{2nd})^4+1(\text{2nd})^5$$

The expansion will be

$$(2x^2y+3xy^2)^5 = 1(2x^2y)^5+5(2x^2y)^4(3xy^2)+10(2x^2y)^3(3xy^2)^2+10(2x^2y)^2(3xy^2)^3+5(2x^2y)(3xy^2)^4+1(3xy^2)^5$$

Then clean up each piece. Simplify those exponents!

$$= 2^5x^{10}y^5 + (5)(2^4)x^8y^4(3xy^2) + 10(2^3)x^6y^3(3^2)x^2y^4 + 10(2^2)x^4y^2(3^3)x^3y^6 + 5(2)x^2y(3^4)x^4y^8 + (3^5)x^5y^{10}$$

Then clean up each piece.

Final answer: $(2x^2y+3xy^2)^5 = 32x^{10}y^5 + 240x^9y^6 + 720x^8y^7 + 1080x^7y^8 + 810x^6y^9 + 243x^5y^{10}$